Wave Mixing in Graphene Resonators Utilizing Coupled-Mode Theory

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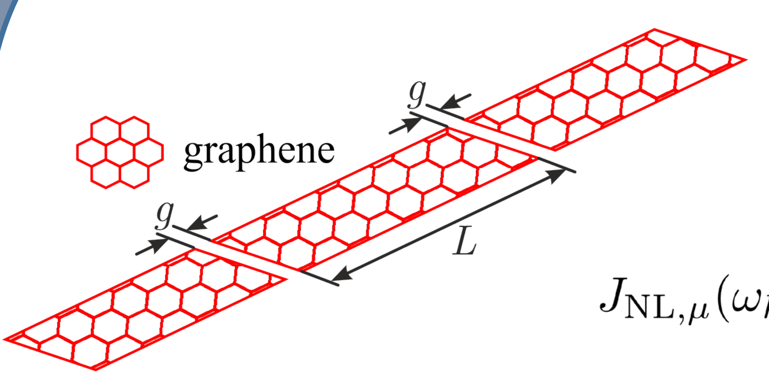
I Introduction

- **Objectives**
  - Demonstrate **efficient** guided wave components for **frequency conversion** in the **unexplored THz frequencies**, were tunable and agile sources are scarce;
  - Present a **solid and accurate** mathematical framework, capable of computationally handling **four-wave mixing** in resonant structures comprising **2D conductive materials** (e.g. **graphene**);
  - Examine the potential for **high Conversion-Efficiency (CE)** wave mixing in the **THz frequency regime**.
- **Motivation**
  - Exploit the capability of **resonant structures** to provide **high CE**, accompanied by the highly nonlinear response of **graphene**, resulting in **low feeding power requirements**.
- **Modelling and Design**
  - Utilize **Coupled-Mode Theory (CMT)** and **Perturbation Theory** for modelling complex nonlinear resonators;
  - Design **compact**, **high-CE**, and with **energy efficient** graphene resonators for DFWM.

II Physical System and Methods

Physical Model



- **Graphene** is modelled as an infinitesimally thin layer, supporting a surface current density with linear and nonlinear parts [1].

$$\mathbf{J}_{\mathrm{lin}} = \sigma_1 \mathbf{E}_{\parallel}$$

$$J_{\mathrm{NL},\mu}(\omega_k + \omega_\ell + \omega_m) = \frac{1}{4}\sum_{\alpha\beta\gamma} \sigma^{(3)}_{\mu\alpha\beta\gamma} E_{k,\alpha} E_{\ell,\beta} E_{m,\gamma}$$

Perturbation Theory

- Excited with highly confined SPPs, **graphene** induces **frequency shifts** in resonant structures due to its **nonlinearity**, estimated using perturbation theory [2,3].
- Shaded term represents an **extra energy contribution**, stored in dispersive graphene [4].

$$\Delta\omega_k = j\frac{\displaystyle\int \mathbf{J}_{\mathrm{NL}}\cdot\mathbf{E}_k^* \mathrm{d}^{d-1}r}{\displaystyle\int \varepsilon_0 \frac{\partial\{\omega\bar{\varepsilon}_r(\omega)\}}{\partial\omega}\mathbf{E}_k\cdot\mathbf{E}_k^*\mathrm{d}^d r + \int \mu_0 \mathbf{H}_k\cdot\mathbf{H}_k^*\mathrm{d}^d r + \boxed{\displaystyle\int \frac{\partial\bar{\sigma}^{(1)}_{\mathrm{Im}}(\omega)}{\partial\omega}\mathbf{E}_k\cdot\mathbf{E}_k^*\mathrm{d}^{d-1}r}}$$

- CMT-friendly version, representing **SPM**, **XPM**, and **DFWM** contributions, assuming interactions of **three waves**:

$$\Delta\omega_k a_k = -\gamma_{kk}|a_k|^2 a_k - 2\gamma_{k\ell}|a_\ell|^2 a_k - 2\gamma_{km}|a_m|^2 a_k - \Phi_k \qquad \Phi_k = \begin{cases} 2\beta_1 a_1^* a_2 a_3, & k=1 \\ \beta_2 a_1^2 a_3^*, & k=2 \\ \beta_3 a_1^2 a_2^*, & k=3 \end{cases}$$

Coupled-Mode Theory

- **CMT** allows to model a complex **nonlinear** resonant system using first order differential equations, parametrized using *linear* FEM eigenvalue simulations [3].

$$\frac{\mathrm{d}a_k}{\mathrm{d}t} = j\left(\omega_k - \gamma_{kk}|a_k|^2 - 2\gamma_{k\ell}|a_\ell|^2 - 2\gamma_{km}|a_m|^2\right)a_k - j\beta_k\Phi_k - \left(\frac{1}{\tau_{i,k}} + \frac{1}{\tau_{e,k}}\right)a_k + \mu_k s_{\mathrm{in,k}}$$

Framework Validation (2D equivalent)

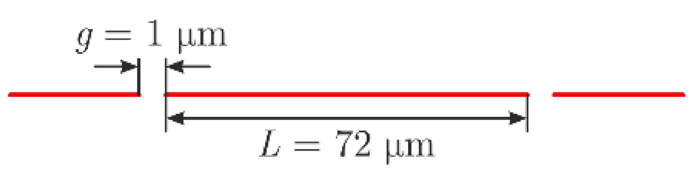
Nonlinear Full-Wave

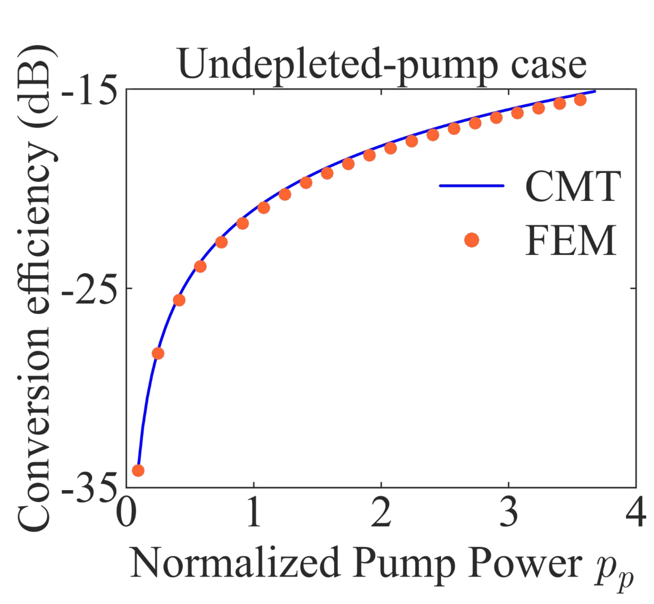
- **Step 1:** Linear simulations @ ω_1 and ω_2.
- **Step 2:** Compute induced surface current @ $\omega_3 = 2\omega_1 - \omega_2$ due to DFWM.
- **Step 3:** Use step 2 surface current as a source for the simulation @ ω_3.
- **Drawback:** This approach cannot take into account the "reverse" interactions between ω_1/ω_3 and ω_2/ω_3 **(undepleted-pump case)**.

Nonlinear CMT

- **Step 1:** Linear finite-element eigenvalue simulations to calculate ω, γ, β, and Q of three consecutive modes.
- **Step 2:** Solution of the nonlinear CMT DEs system with $\beta_2 = \beta_3 = 0$ **(undepleted-pump case)**.

Simulated 2D System





- Excellent agreement, **validating the accuracy** of the demonstrated framework.

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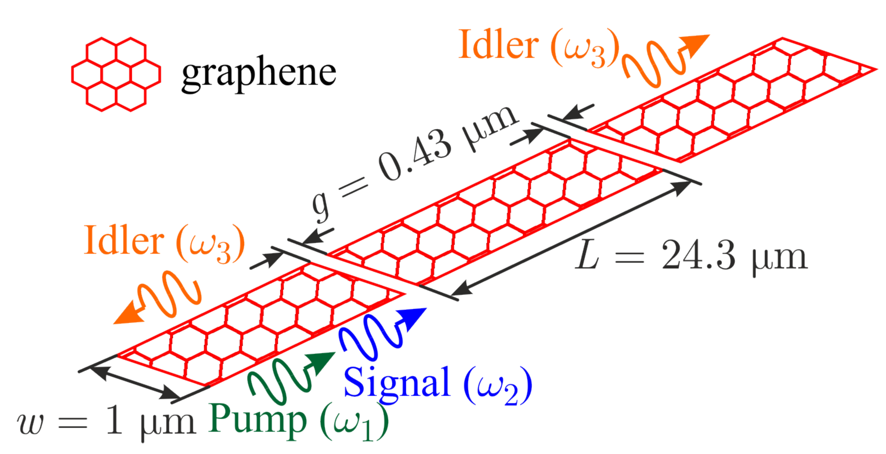
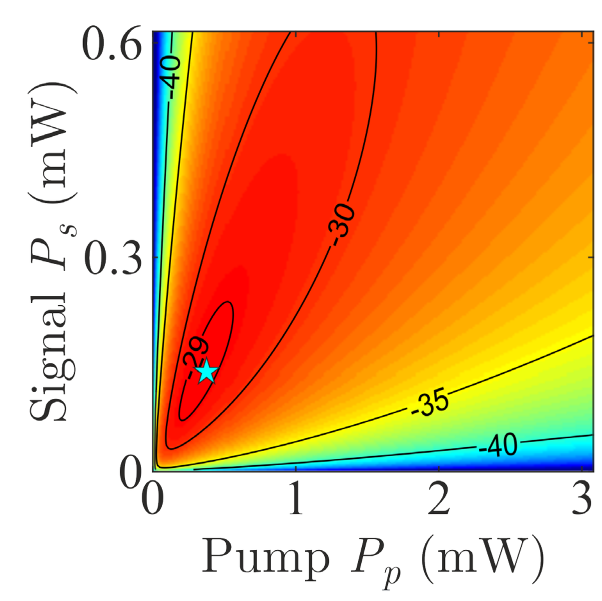
III Results

3D Graphene Nanoribbon Resonator for DFWM in the THz

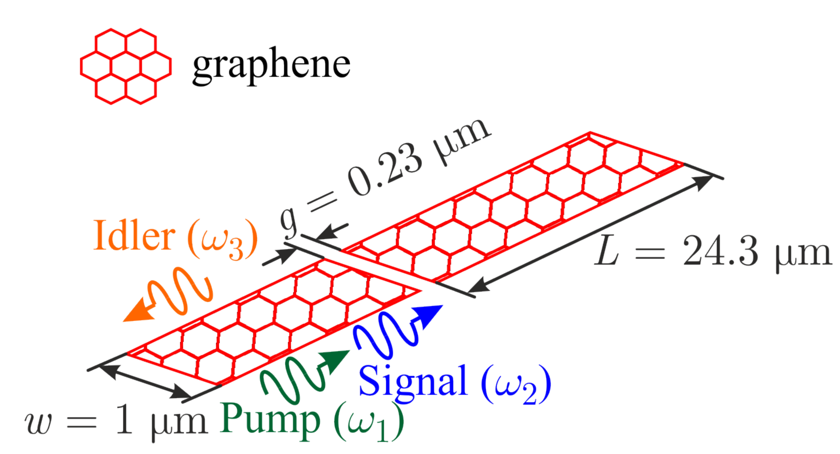
Double Waveguide Configuration

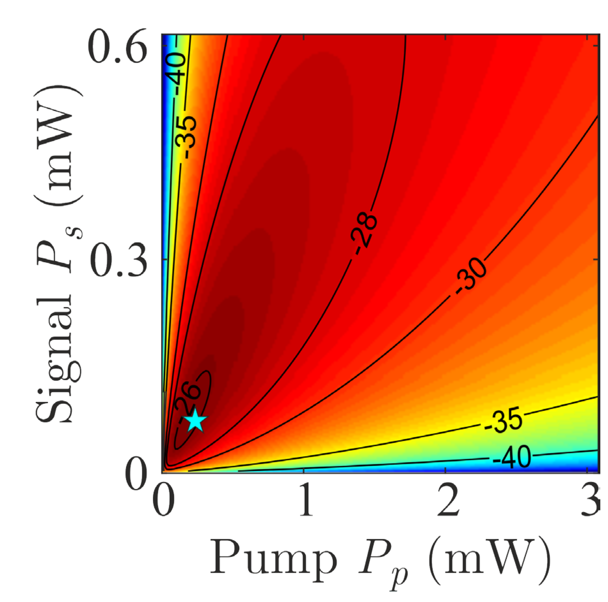




CE_{max} = -28.8	$P_{in,p}$ = 311 μW	
	$P_{in,s}$ = 126 μW	
m = 7 (idler)	m = 8 (pump)	m = 9 (signal)
f_3 = 4.53 THz	f_1 = 5.00 THz	f_2 = 5.45 THz
Q_i = 1097	Q_i = 1234	Q_i = 1338
Q_e = 222	Q_e = 280	Q_e = 343

Single Waveguide Configuration

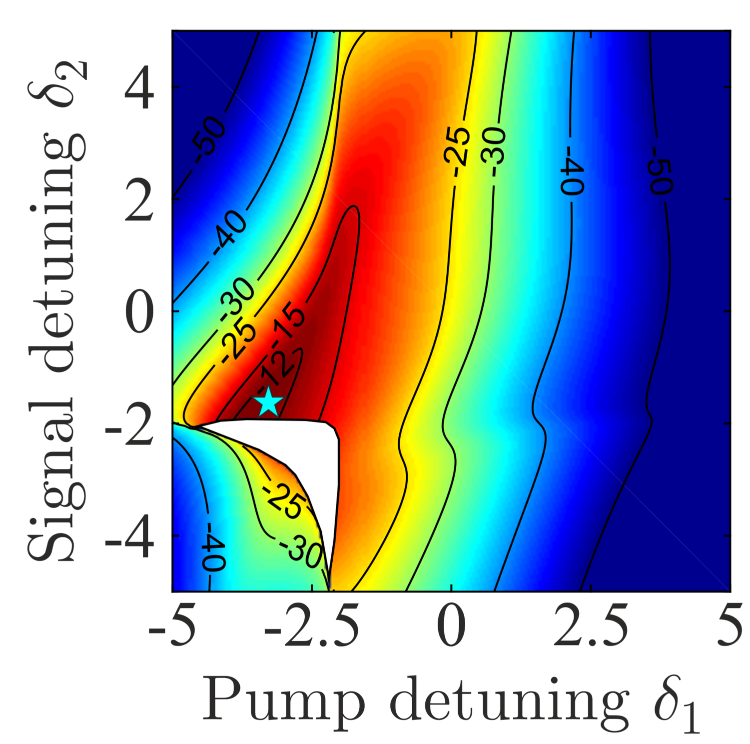




CE_{max} = -25.7	$P_{in,p}$ = 172 μW	
	$P_{in,s}$ = 71 μW	
m = 7 (idler)	m = 8 (pump)	m = 9 (signal)
f_3 = 4.53 THz	f_1 = 5.00 THz	f_2 = 5.45 THz
Q_i = 1094	Q_i = 1231	Q_i = 1334
Q_e = 221	Q_e = 265	Q_e = 314

- **High Conversion Efficiency** for **low power (< mW)** pump and signal.
- **Single waveguide** configuration is **advantageous** (+3 dB in CE), due to the nature of the resonant system configuration.
- **SPM and XMP are included**, shifting the resonance frequencies ω_k away from the operating frequencies ω_k^{op}.
- Even **higher CEs** can be achieved by **pre-shifting** the operating frequencies [5].

Kerr-Induced Frequency Shift Compensation



- Pre-shifting the operating frequencies, **partially compensates** the effect of SPM and XPM (resonance frequencies shift).
- An **important boost in CE** can be easily achieved.
- Normalized shifts of δ_1 = -3.6 and δ_2 = -1.9 result in a **further 14 dB boost** for the optimal single output configuration.
- **Overall CE** is as high as **-11.3 dB**.
- **Further improvements** (a few more dBs) can be introduced by **fine-tuning** of the input pump and signal power.
- **Optical bistability** appears for negative δ_1 and δ_2 (white area), due to SPM and XPM interactions.

IV Conclusions

A practical **graphene nanoribbon resonator**, manifesting DFWM in the unexplored THz region where tunable and agile sources are scarce, is **analyzed and designed**. The proposed resonant system exhibits a **high CE** of **-11.3 dB**, with low power requirements of **172** and **71 μW** for the **pump** and **signal waves**, respectively. The analysis is based on a **rigorous mathematical framework** for modelling **wave mixing** in resonant systems involving nonlinear **2D materials**. The framework is systematically validated, **proven highly accurate**.

V References

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