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Rigorous Quality Factor Calculation in Contemporary Optical Resonant Systems

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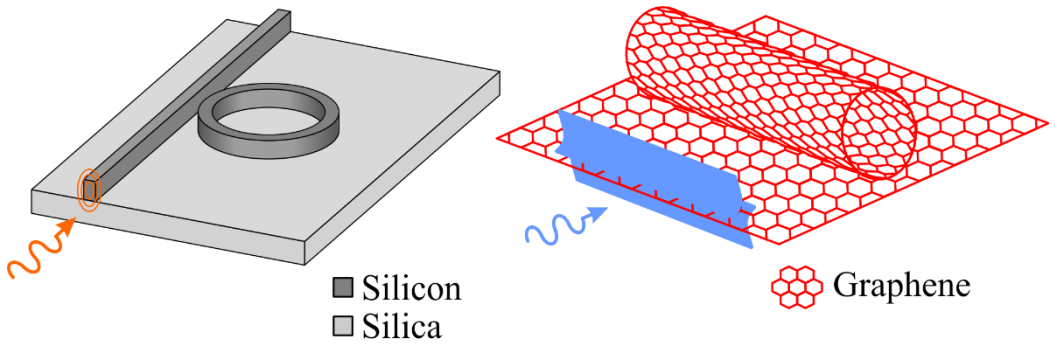
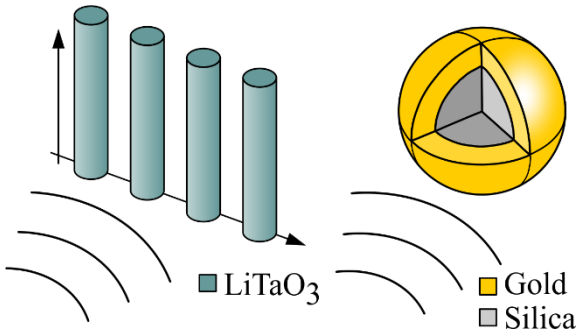
Motivation

- Resonant structures are ubiquitous in modern photonics integrated circuits
- Contemporary platforms (plasmonics) and materials (graphene) exhibit distinct characteristics
- Quality factor calculation is non-trivial





Objectives

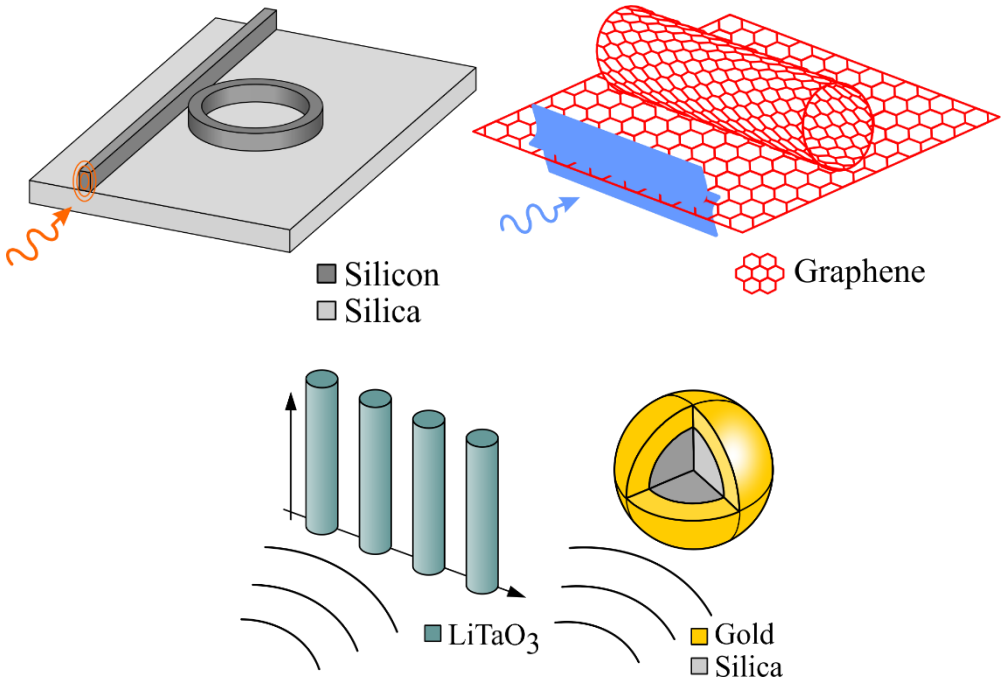
- Collectively present the *Q*-factor calculation techniques using commercially available software implementing FEM (COMSOL Multiphysics, ANSYS HFSS, CST Microwave studio)
- Highlight the effect of material dispersion, ohmic loss and light leakage on the calculations
- Propose alternatives routes

Presentation outline

- **Computational methods for calculating the Quality Factor**
  1. Eigenvector
  2. Field distribution
  3. Eigenfrequency
  4. Spectral response

- **Application range of the methods via examples**
  - Silicon ring resonator at the NIR
  - Graphene tube resonator at the FIR
  - Polaritonic rod at the FIR
  - Plasmonic core-shell at the visible

- **Conclusions**



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Computational methods for calculating the Quality Factor

Quality factor definition (1): *the eigenmode and the field distribution methods*

Quality Factor definition

$$\boxed{Q = \omega_0 \frac{W}{P_{\text{loss}}}}$$

- W Stored energy on resonance
- P_{loss} Power loss

$$P_{\text{loss}} = \{P_{\text{res}}, P_{\text{rad}}, P_i, P_e, P_\ell\}$$

- Resistive
- Radiation

} Intrinsic

- External (coupling)

} Loaded

[D. M. Pozar, *Microwave Engineering*]

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Resistive, Radiation: Intrinsic; Resistive, Radiation, External (coupling): Loaded

[D. M. Pozar, *Microwave Engineering*]

Energy and Losses calculation

$$W = \frac{1}{4}\int_V \varepsilon_0 \frac{\partial\{\omega\varepsilon_r(\omega)\}}{\partial\omega}|\mathbf{E}|^2 \mathrm{d}r + \frac{1}{4}\int_V \mu_0|\mathbf{H}|^2\mathrm{d}r + \frac{1}{4}\int_V \frac{\partial \mathrm{Im}\{\sigma(\omega)\}}{\partial\omega}|\mathbf{E}|^2\mathrm{d}r$$

Material dispersion

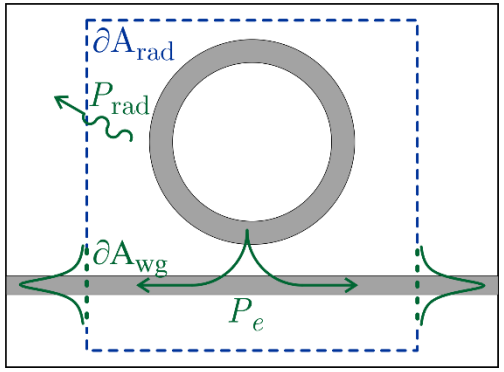
$$P_{\text{res}} = \frac{1}{2}\int_V \mathrm{Re}\{\mathbf{E}\cdot\mathbf{J}^*\}\,\mathrm{d}r = \frac{1}{2}\int_V \mathrm{Re}\{\sigma\}|\mathbf{E}|^2\mathrm{d}r + \frac{1}{2}\int_V \omega\varepsilon_0 \mathrm{Im}\{\varepsilon_r\}|\mathbf{E}|^2\mathrm{d}r$$

Ohmic loss

Dielectric loss

$$P_{\text{rad}} = \int_{\partial V} \mathrm{Re}\{\mathbf{S_c}\}\cdot\hat{\mathbf{n}}\,\mathrm{d}r = \frac{1}{2}\int_{\partial V} \mathrm{Re}\{\mathbf{E}\times\mathbf{H}^*\}\cdot\hat{\mathbf{n}}\,\mathrm{d}r$$

$$P_e = \int_{\text{wg}} \mathrm{Re}\{\mathbf{S_c}\}\cdot\hat{\mathbf{n}}\,\mathrm{d}r = \frac{1}{2}\int_{\text{wg}} \mathrm{Re}\{\mathbf{E}\times\mathbf{H}^*\}\cdot\hat{\mathbf{n}}\,\mathrm{d}r$$

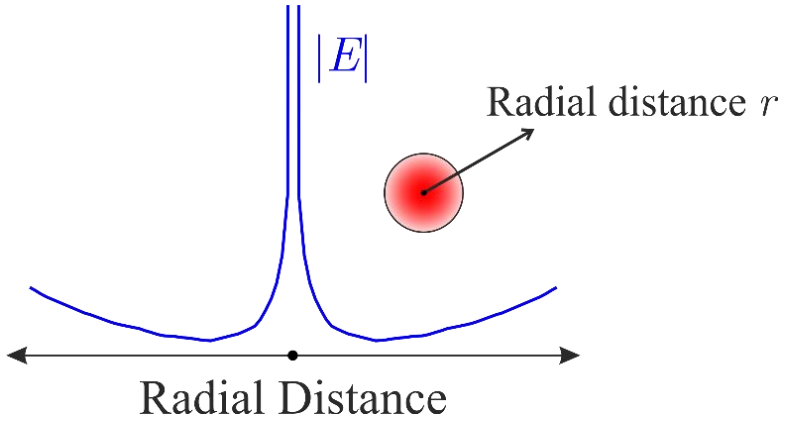


$$\frac{1}{Q_\ell} = \frac{1}{Q_i} + \frac{1}{Q_e} = \frac{1}{Q_{\text{res}}} + \frac{1}{Q_{\text{rad}}} + \frac{1}{Q_e}$$

Quality factor definition (2): *the eigenmode and the field distribution methods*

1. The *eigenmode* method

- Complex eigenvalue $\tilde{\omega} = \omega' + j\omega''$
- Temporal decay $\propto \exp\{-\omega'' t\}$
- Spatial exponential divergence! $\propto \exp\{+r\omega''/c\}$



- W and P_{loss} diverge but W/P_{loss} is constant

$$P_{\text{rad}} + P_e = 2\omega'' W - P_{\text{res}}$$

[L. D. Landau, *Electrodynamics of continuous media*]

Commercial FEM eigenvalue implementations do not incorporate dispersion

EIGENMOME METHOD FAILS IN HIGHLY DISPERSIVE SYSTEMS

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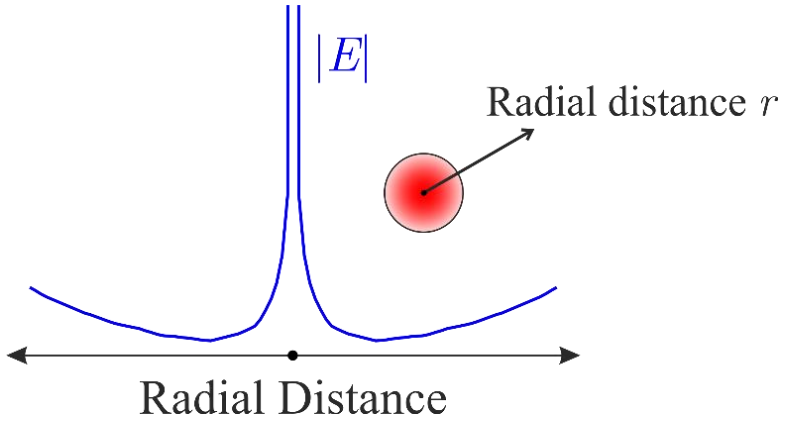

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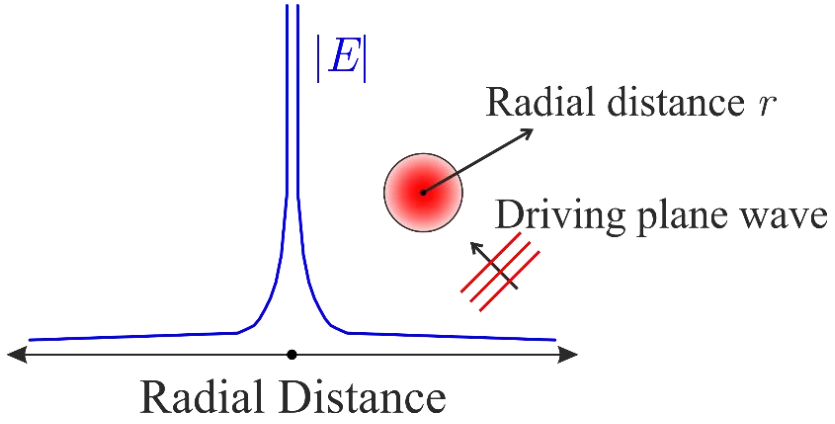
2. The *field distribution* method

- Driven harmonic simulation @ $\omega_0 = \omega'$
- Physically incorporates dispersion
- Spatial and temporal mode decay



- W depends on the integration window
- W/P_{loss} diverges with radius

FIELD DISTRIBUTION METHOD REQUIRES CAREFUL CHOICE OF THE INTEGRATION WINDOW

Alternative approaches: *the eigenfrequency and the spectral response method*

3. The *eigenfrequency* method

- Complex eigenvalue $\tilde{\omega} = \omega' + j\omega''$

$$Q = \frac{\omega'}{2\omega''}$$

Commercial FEM eigenvalue implementations do not incorporate dispersion

EIGENFREQUENY METHOD FAILS IN DISPERSIVE SYSTEMS

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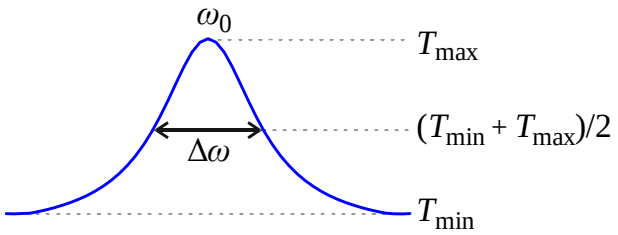
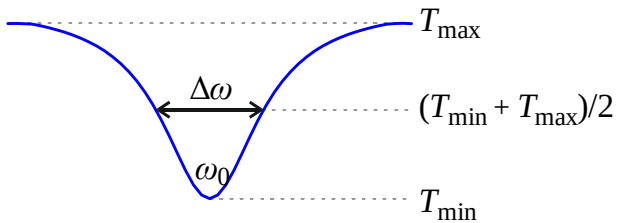
EIGENFREQUENY METHOD FAILS IN DISPERSIVE SYSTEMS

4. The *spectral response* method

- Driven harmonic simulation @ frequencies around ω_0

$$Q = \frac{\omega_0}{\Delta\omega}$$





SPECTRAL RESPONSE METHOD IS ALWAYS CORRECT

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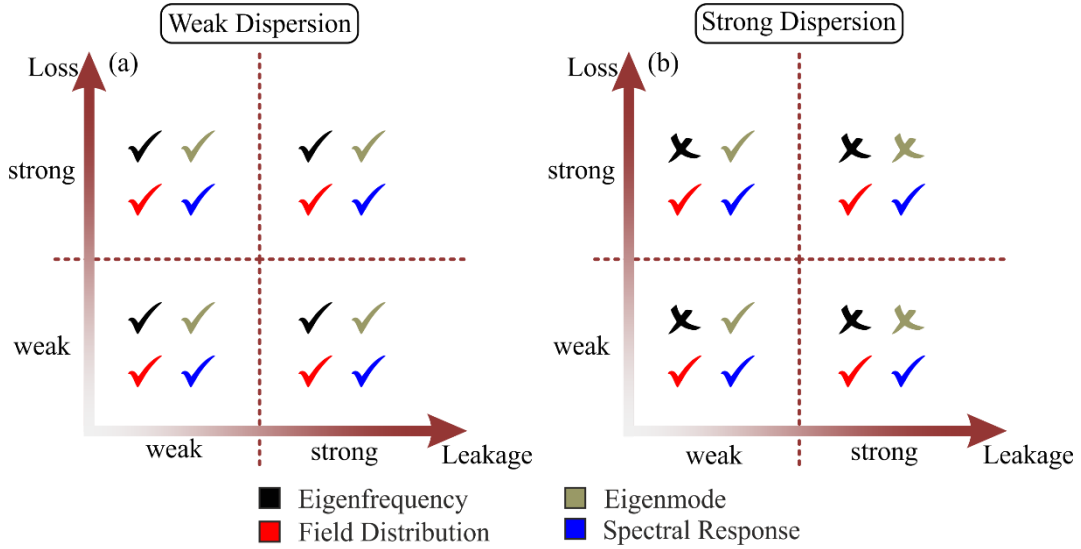

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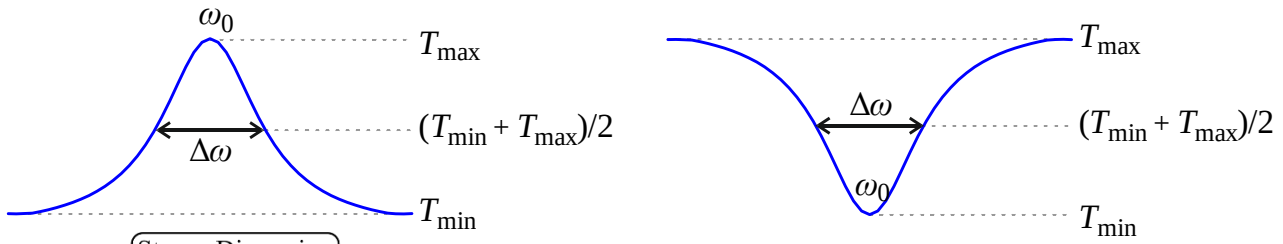

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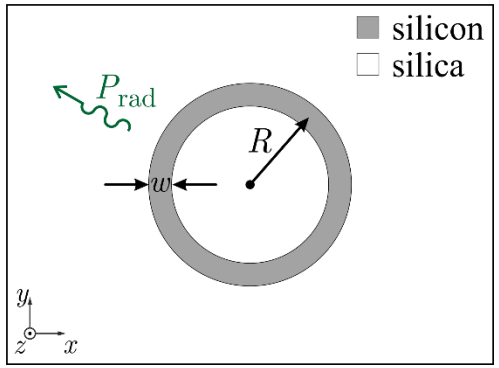
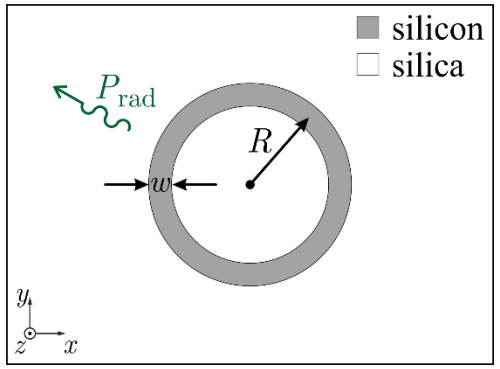
Application range of the methods via examples

Silicon slab ring resonator @ Near Infrared spectrum // Intrinsic Q



- Weak dispersion
- Absence of ohmic loss

Silicon slab ring resonator @ Near Infrared spectrum // Intrinsic Q



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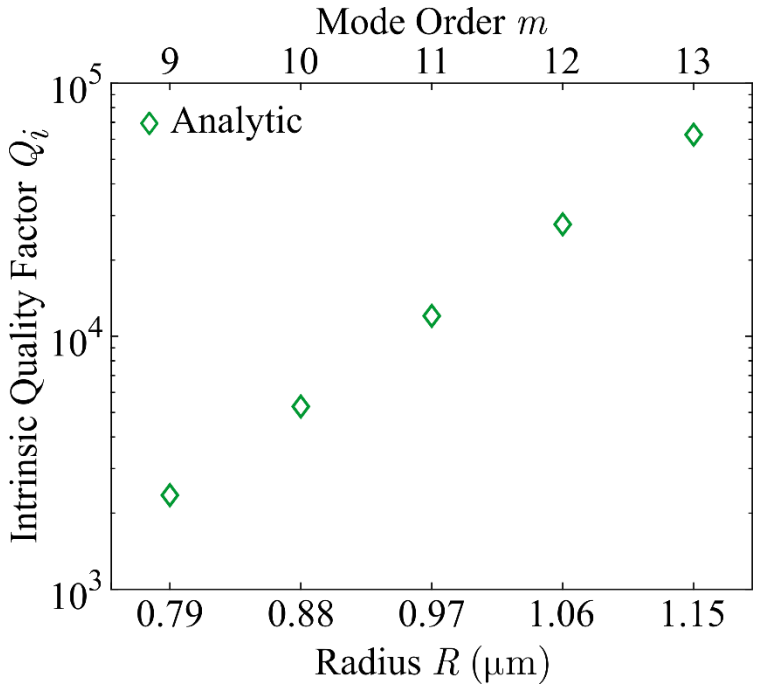
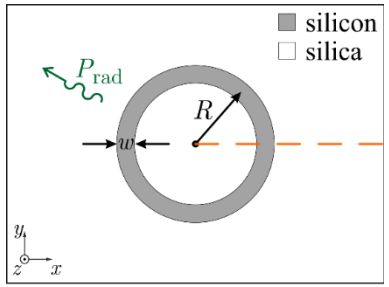
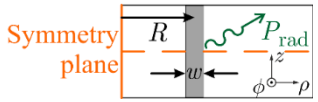


$$Q_i = \frac{\sqrt{a}}{1-a}\frac{(2\pi R)\pi n_g}{\lambda_m}$$



[Bogaerts, *Laser Photonics Rev. 6*, 47, 2012]
[Pendry, *Science 312*, 1780, 2006]



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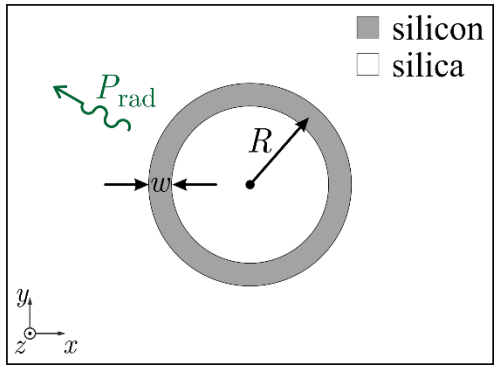

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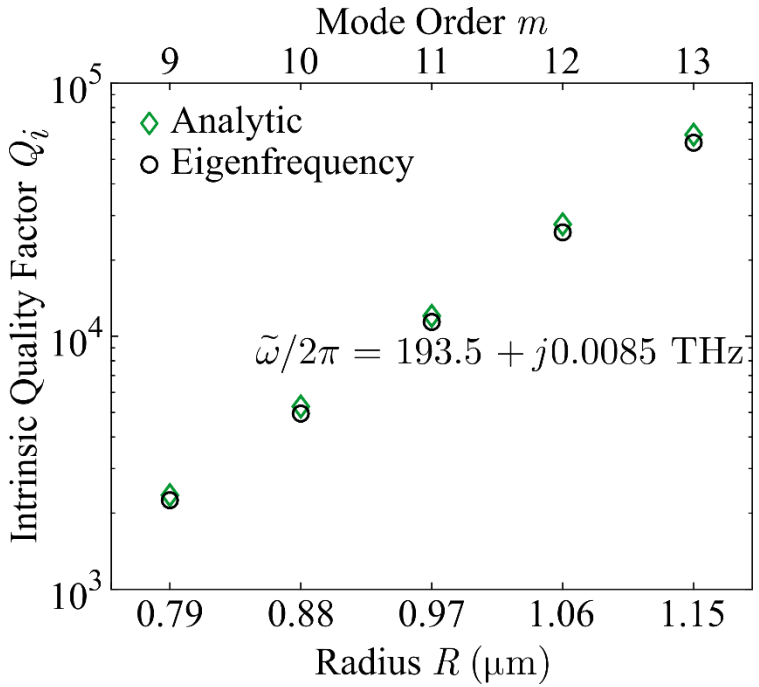
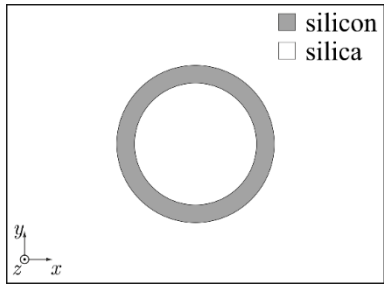


$$Q_i = \frac{\omega'}{2\omega''}$$



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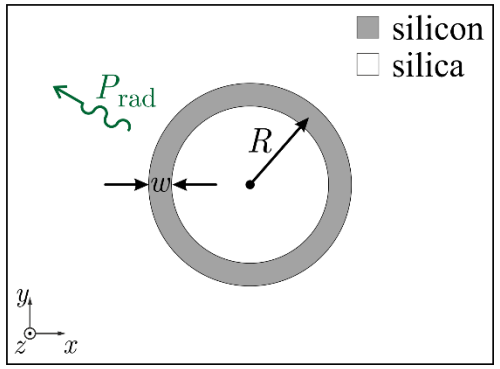

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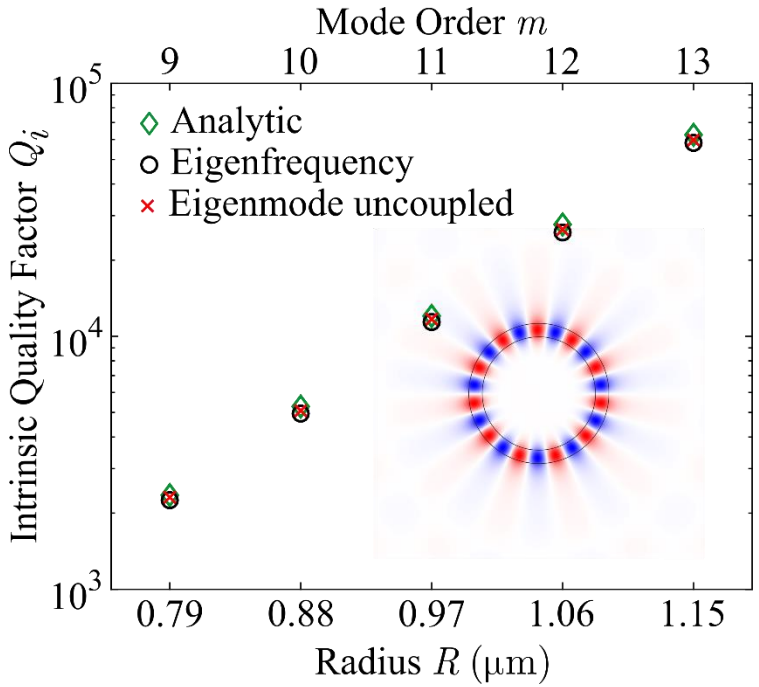
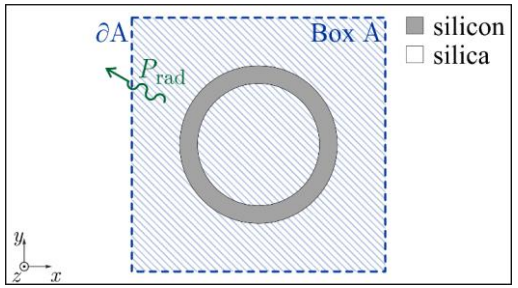
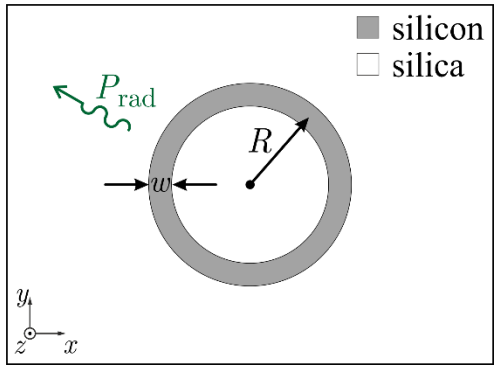


$$Q_i = \omega_0 \frac{W}{P_{\mathrm{rad}}}$$



Silicon slab ring resonator @ Near Infrared spectrum // Intrinsic Q



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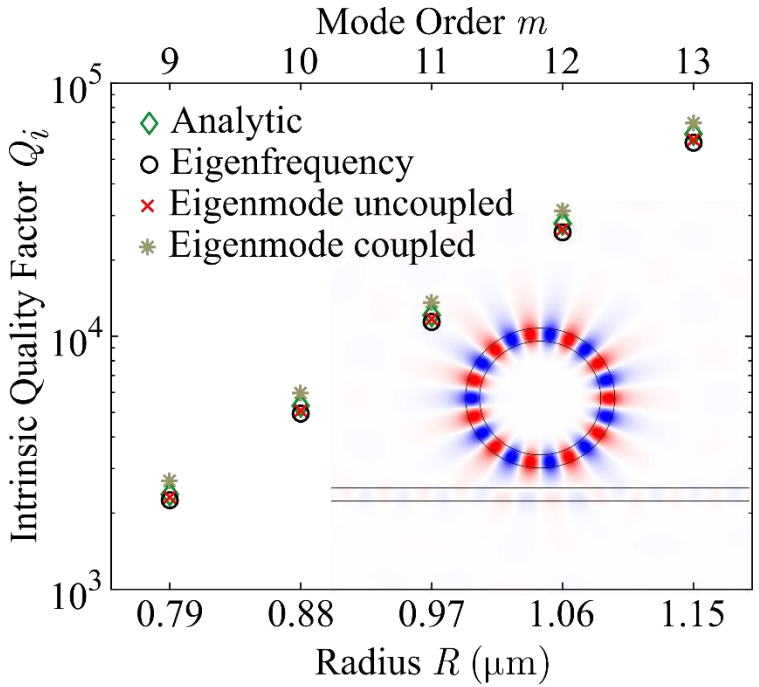
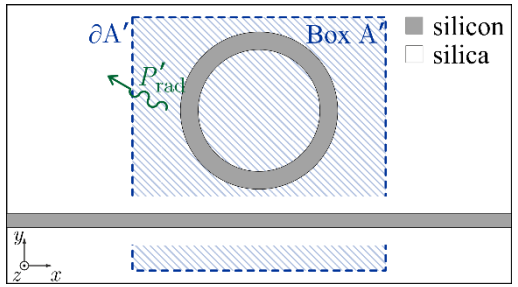


$$Q_i' = \omega_0 \frac{W}{P'_{\text{rad}}}$$



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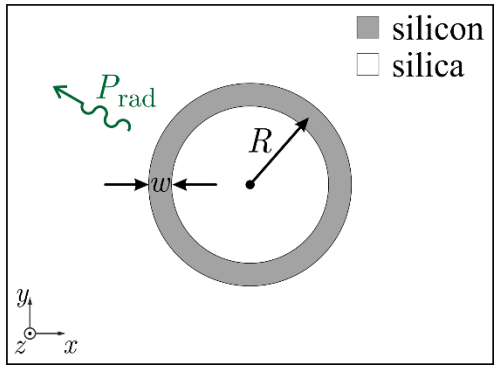

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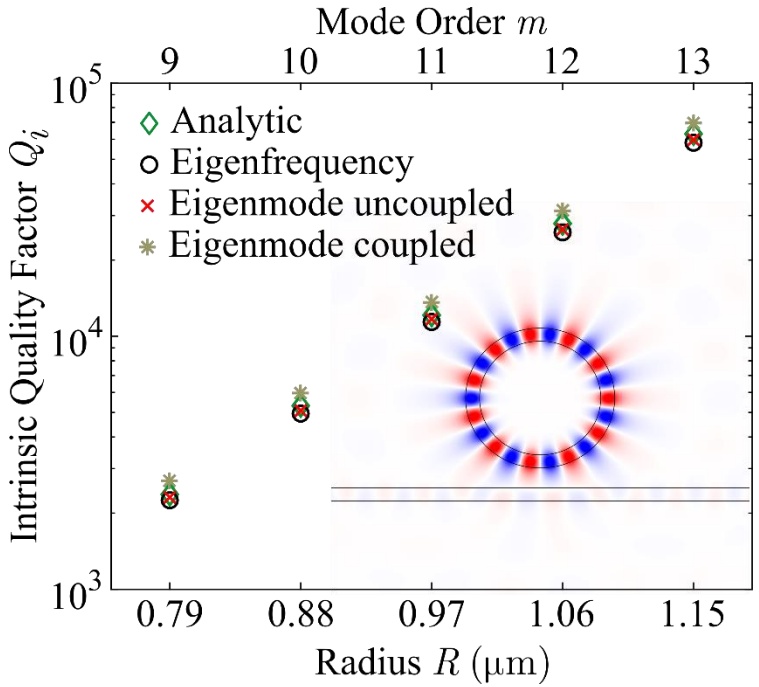

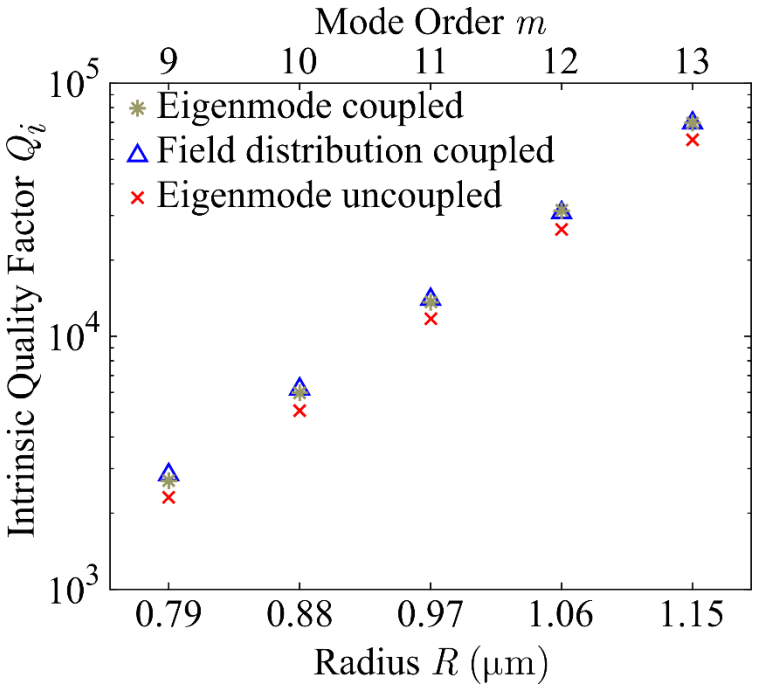

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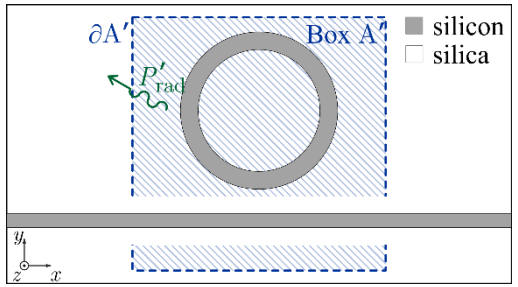
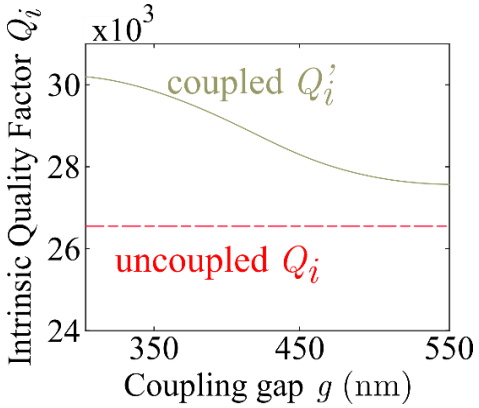
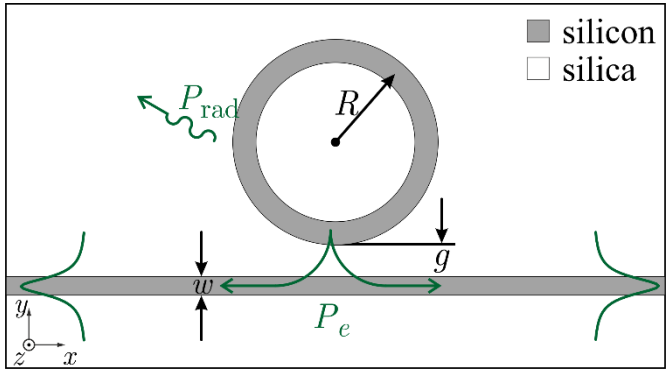




Silicon slab ring resonator @ Near Infrared spectrum // External and Loaded Q



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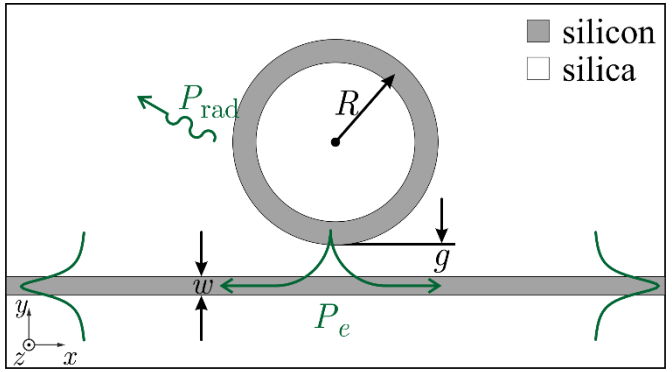

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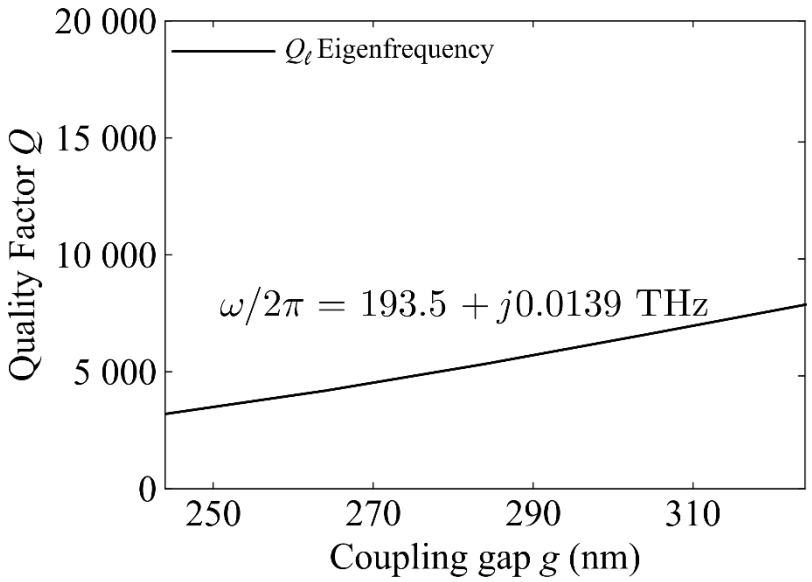

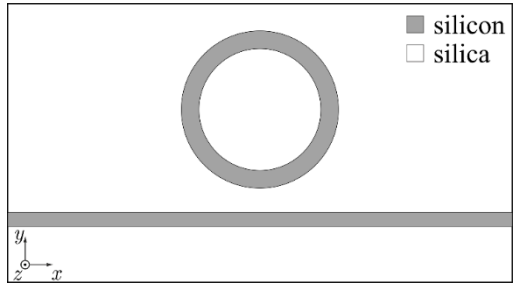

$$Q_\ell = \frac{\omega'}{2\omega''}$$

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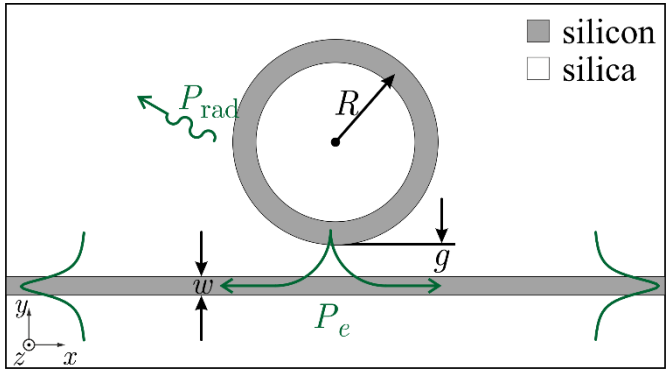

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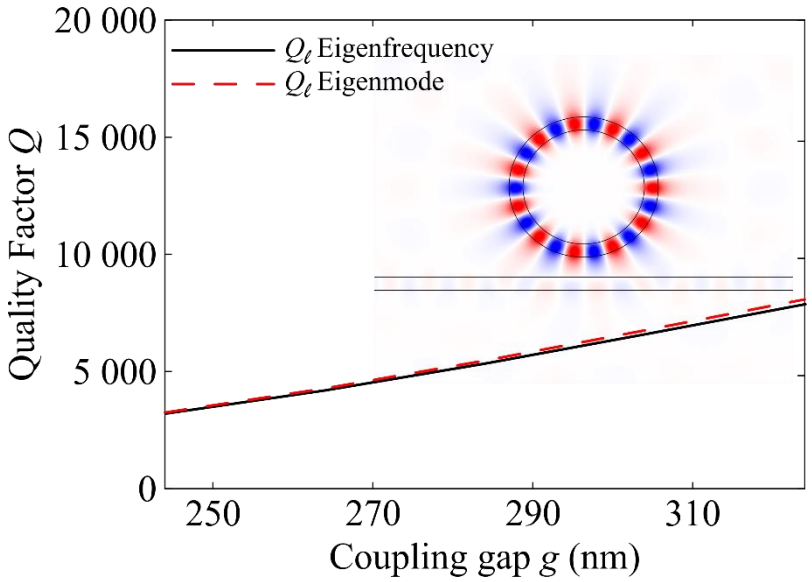

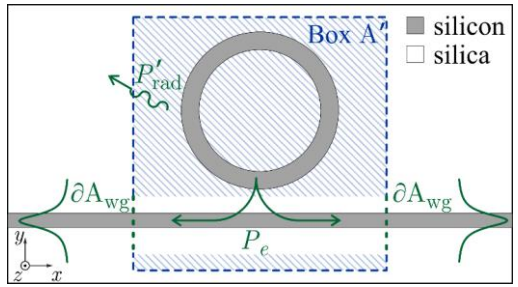

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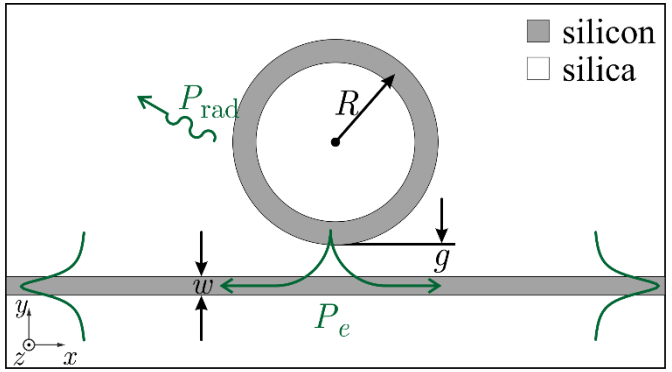

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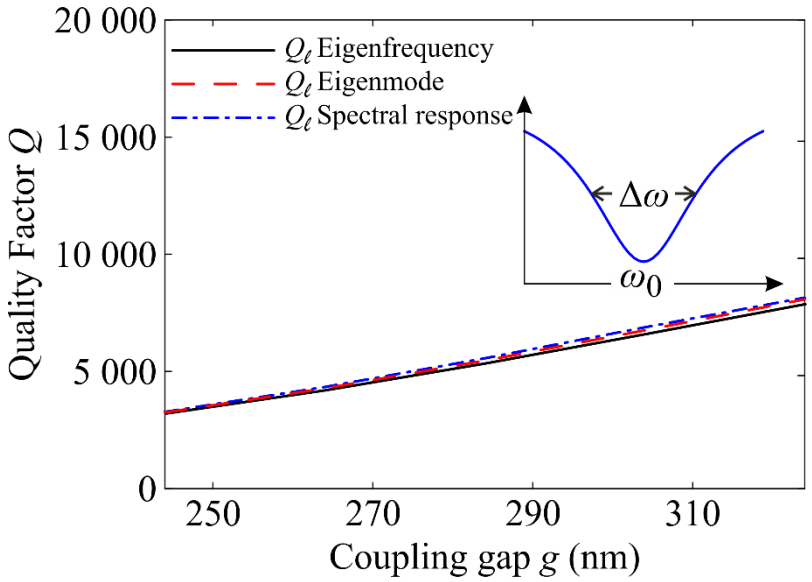

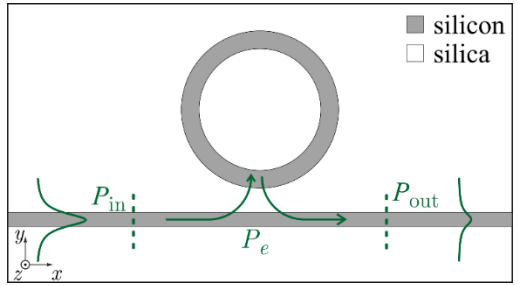

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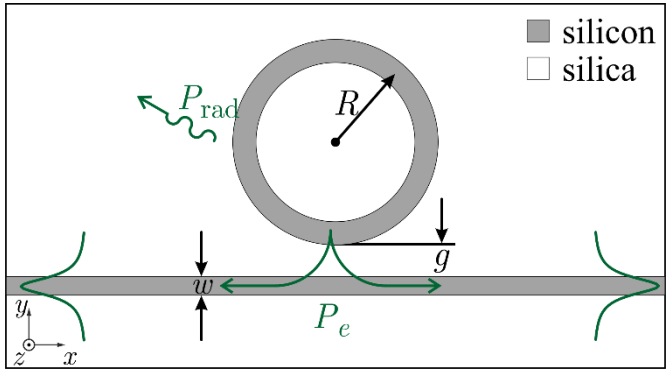

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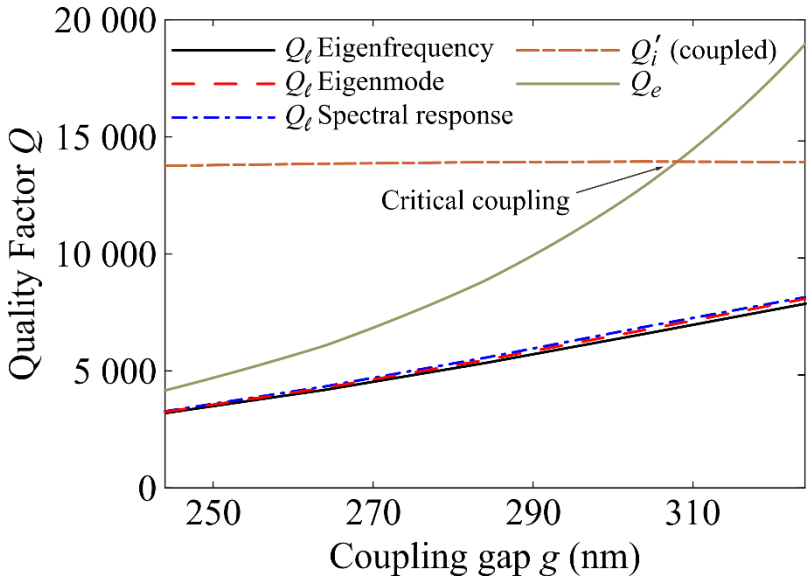
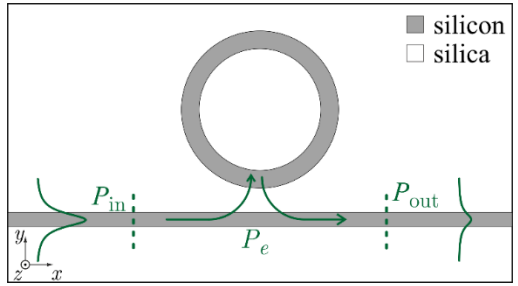
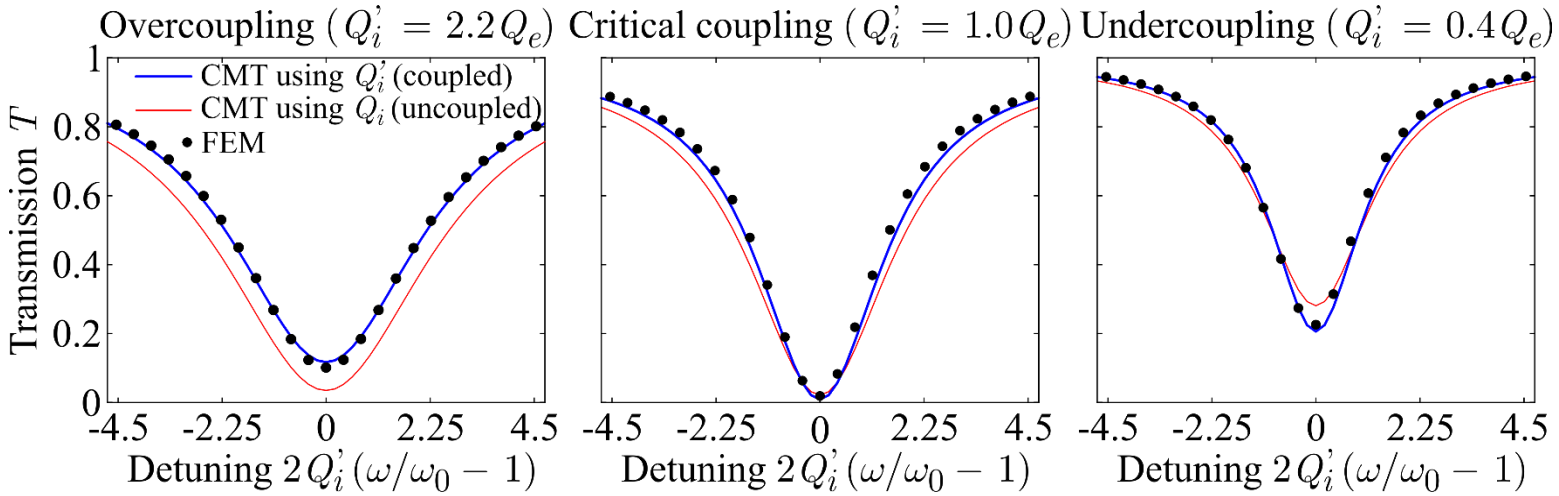




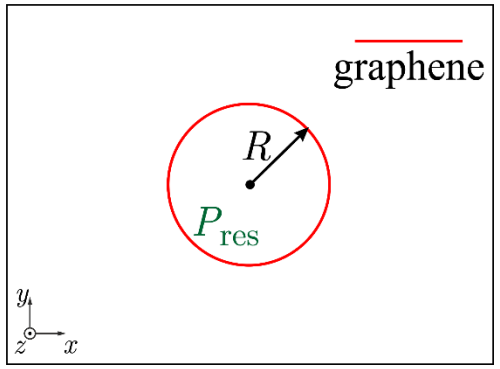
$$Q_\ell = \frac{\omega_0}{\Delta\omega}$$



$$T = \frac{4(\omega/\omega_0 - 1)^2 + (1/Q_i - 1/Q_e)^2}{4(\omega/\omega_0 - 1)^2 + (1/Q_i + 1/Q_e)^2}$$

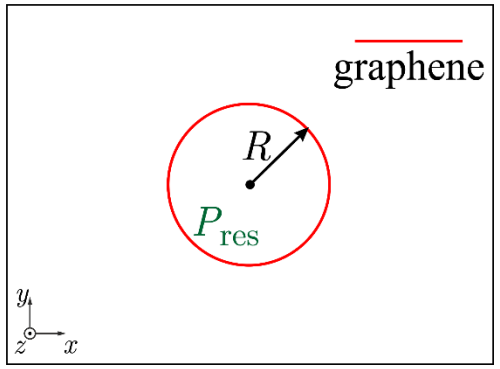
[H. A. Haus, *Waves and Fields in Optoelectronics*]

Graphene tube resonator @ Far Infrared (THz) spectrum



- Strong dispersion
- Absence of radiation

Graphene tube resonator @ Far Infrared (THz) spectrum



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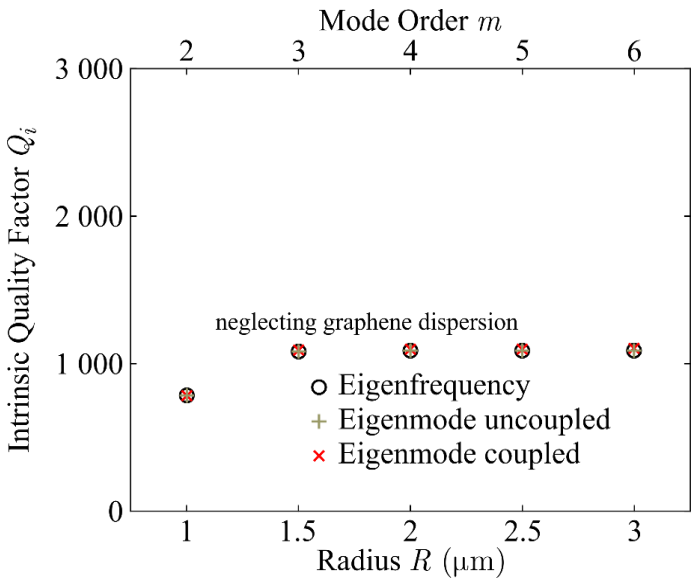
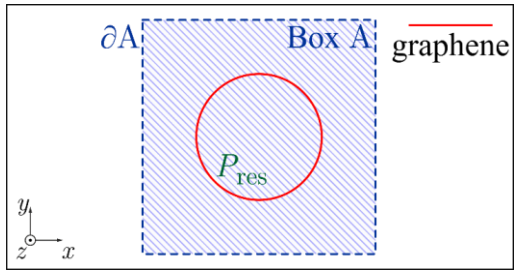
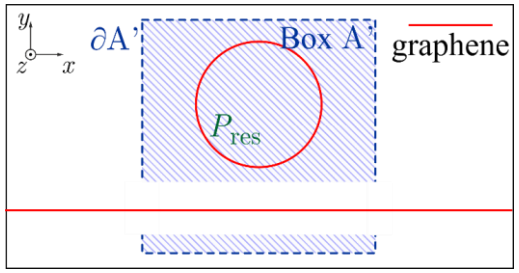


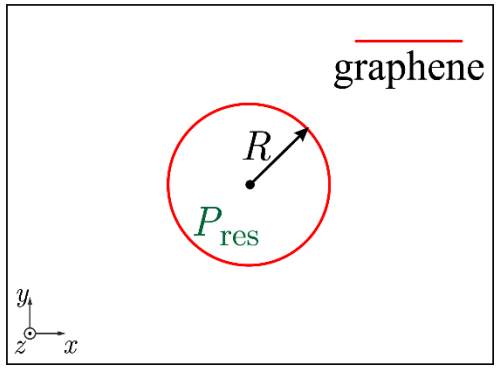




$$Q_i = \omega_0 \frac{W}{P_{\text{res}}}$$

$$W = \frac{1}{4}\int_V \varepsilon_0 \varepsilon_r |\mathbf{E}|^2 \mathrm{d}r + \frac{1}{4}\int_V \mu_0 |\mathbf{H}|^2 \mathrm{d}r$$

Graphene tube resonator @ Far Infrared (THz) spectrum



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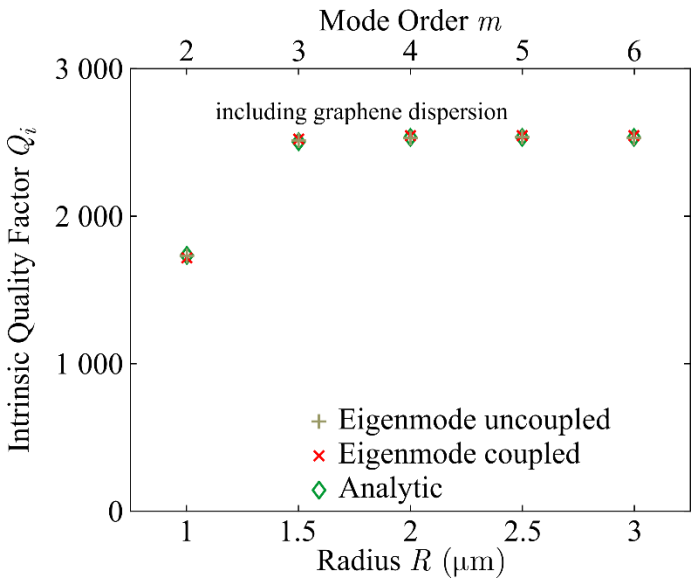

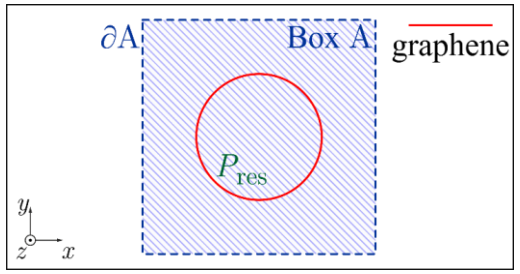

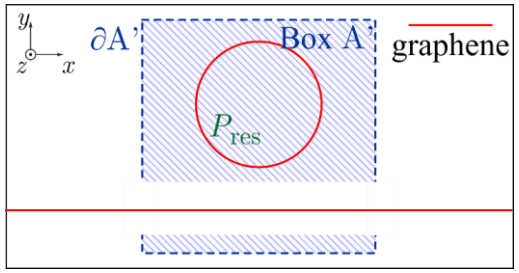

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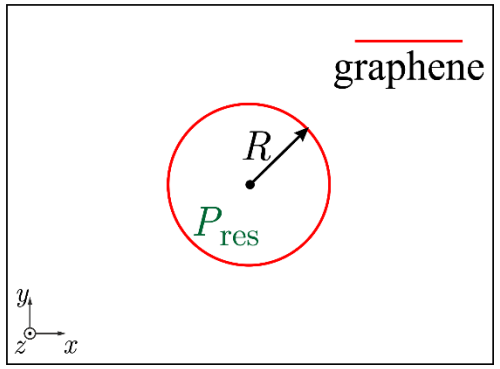
[Christopoulos, *Phys. Rev. E 94*, 062219, 2016]

Graphene tube resonator @ Far Infrared (THz) spectrum



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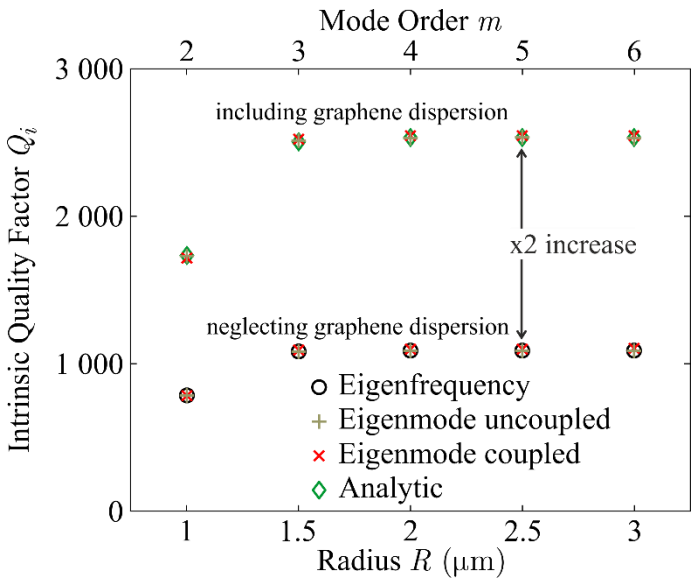

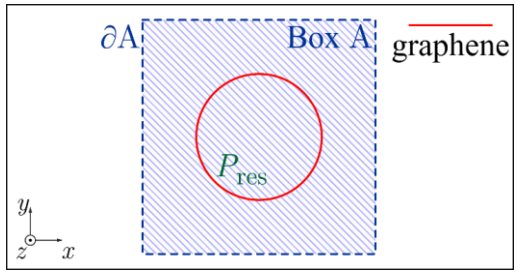

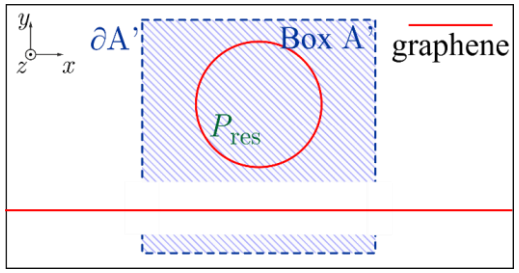

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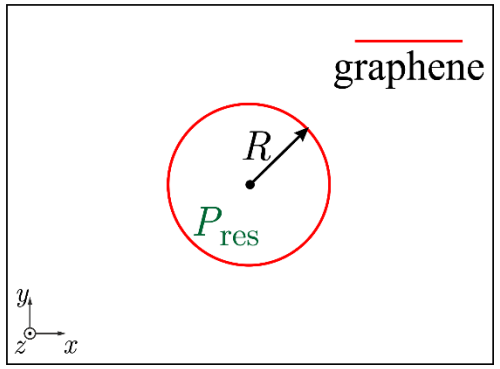
[Christopoulos, *Phys. Rev. E 94*, 062219, 2016]

Graphene tube resonator @ Far Infrared (THz) spectrum



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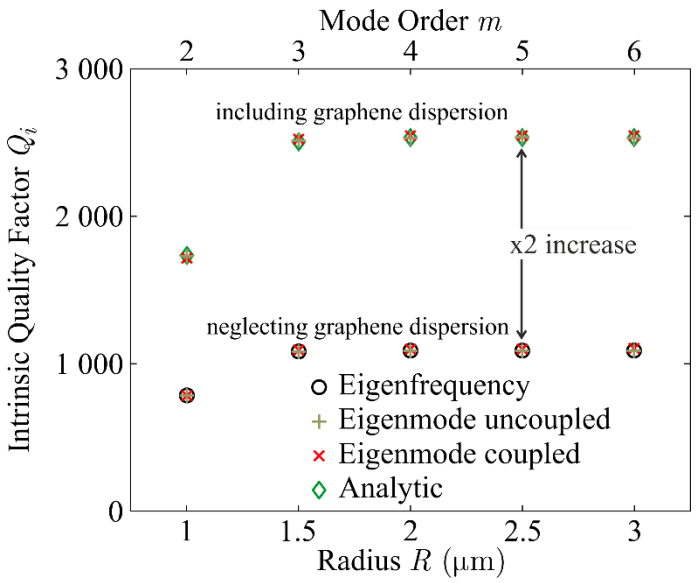

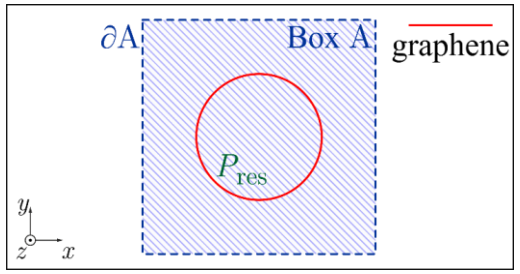

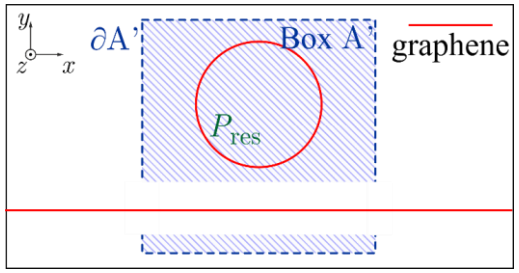

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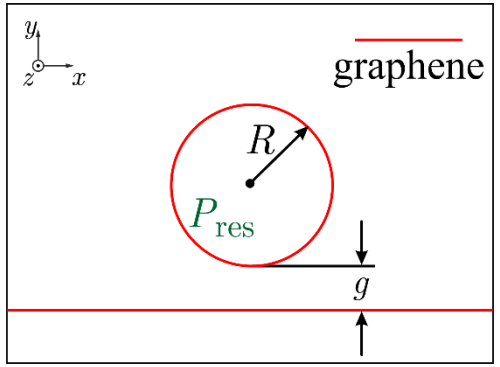
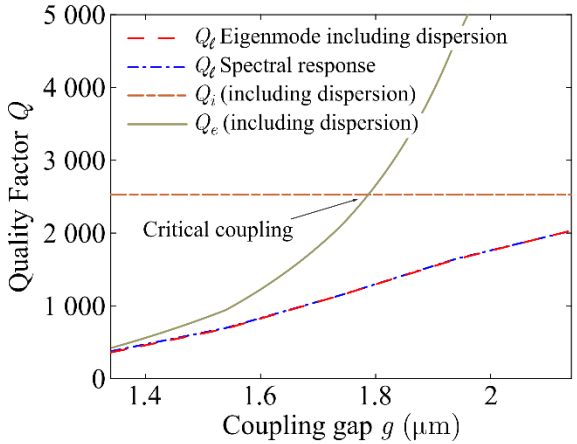
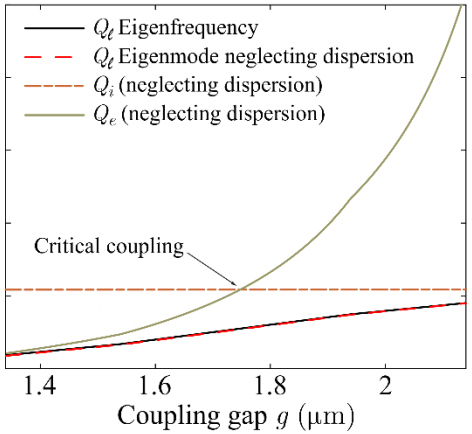
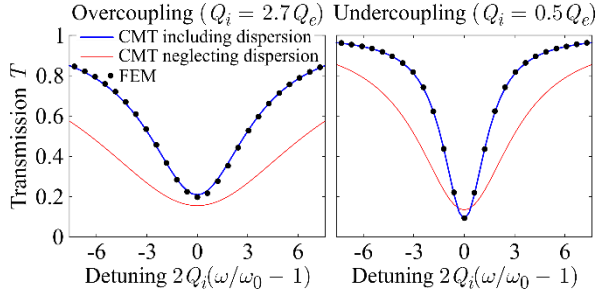
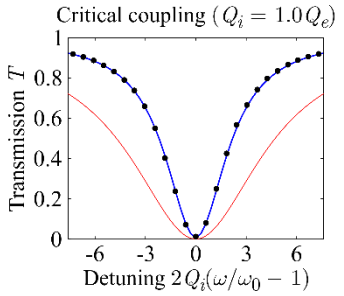
[Christopoulos, *Phys. Rev. E 94*, 062219, 2016]



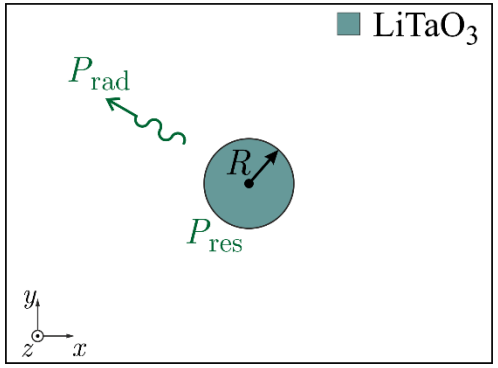






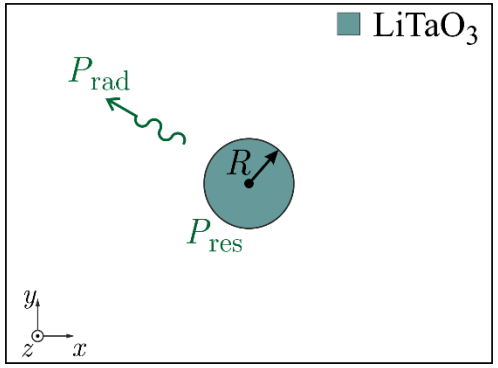


Dielectric rod resonator/metasurface @ Far Infrared (THz) spectrum



- Weak dispersion
- Radiation and ohmic loss

Dielectric rod resonator/metasurface @ Far Infrared (THz) spectrum



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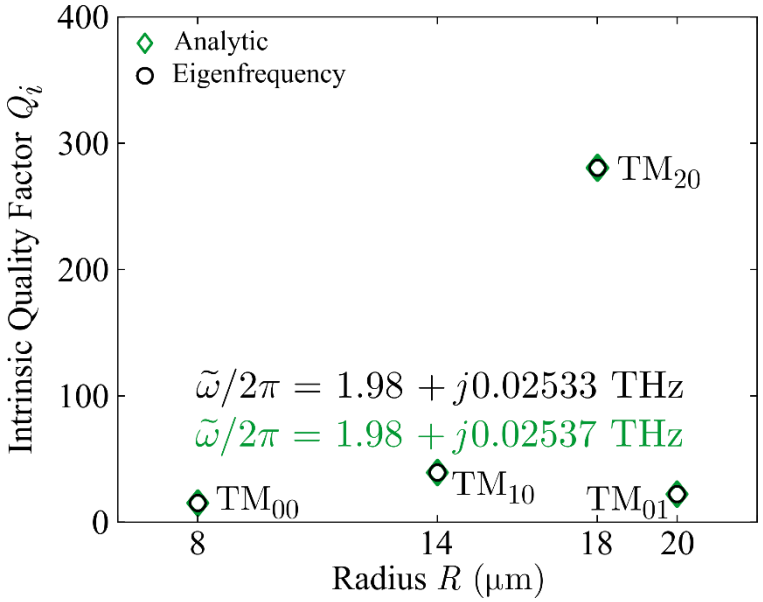
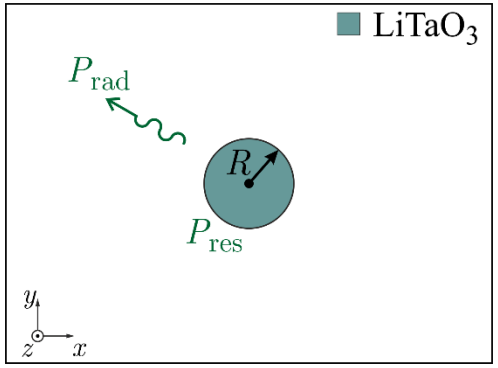


$$Q_i = \frac{\omega'}{2\omega''}$$

$$\upsilon \frac{J_m'(u)}{J_m(u)} = u \frac{H_m^{(1)\prime}(\upsilon)}{H_m^{(1)}(\upsilon)} \qquad u = \tilde{k}_0 n_{\text{LiTaO}_3} R, \quad \upsilon = \tilde{k}_0 R$$

[J. A. Stratton, *Electromagnetic Theory*]

Dielectric rod resonator/metasurface @ Far Infrared (THz) spectrum



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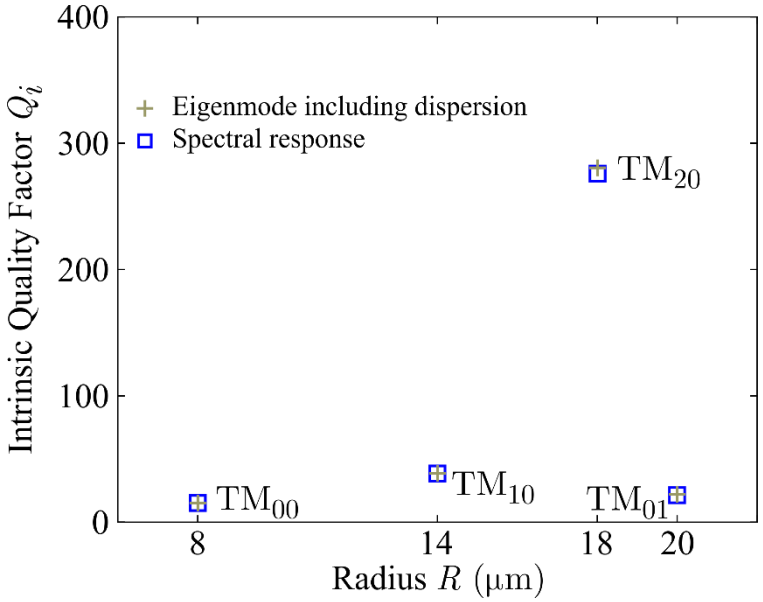

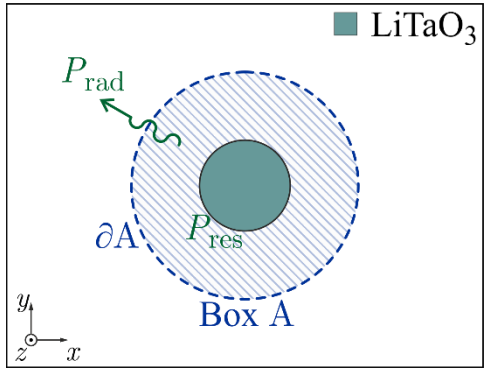

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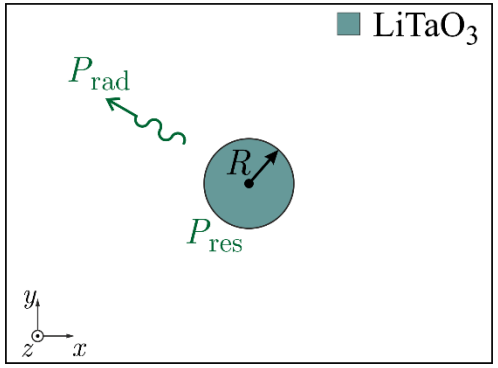

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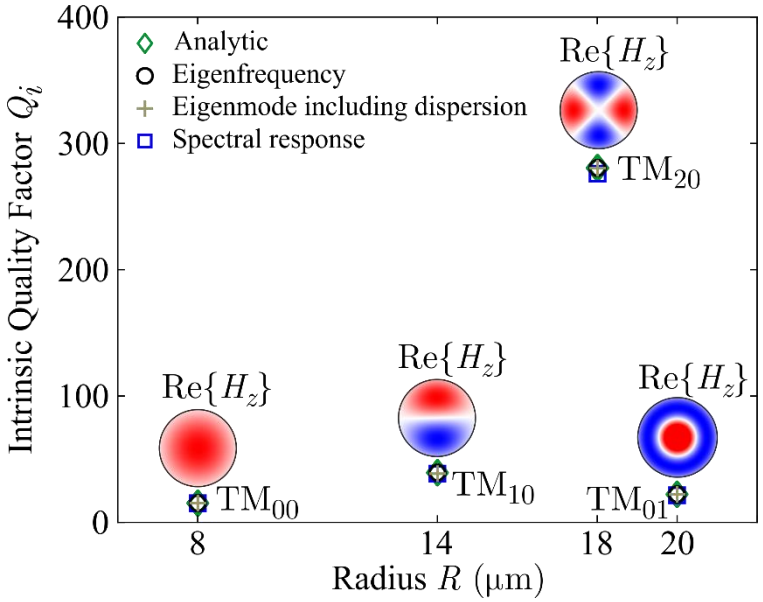
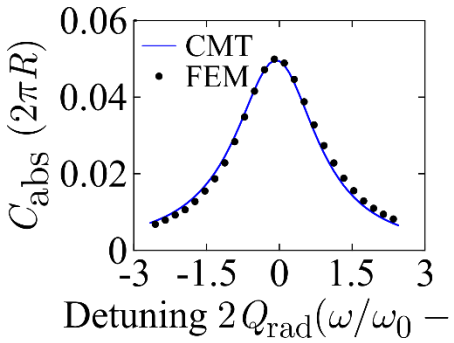
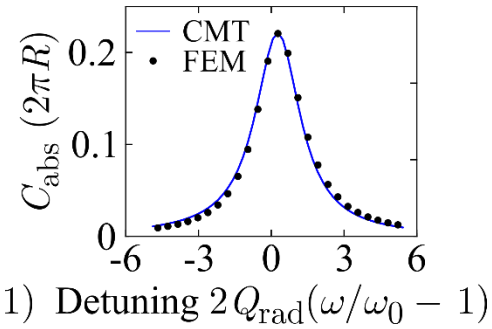


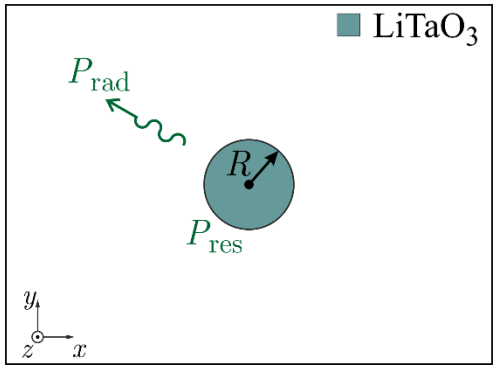




$$C_{abs} = \frac{2(m+1)\lambda}{\pi} \frac{(1/Q_{res})(1/Q_{rad})}{4(\omega/\omega_0-1)^2+(1/Q_{res}+1/Q_{rad})^2}$$

[Ruan, *J. Phys. Chem. C 114*, 013901, 2010]

Dielectric rod resonator/metasurface @ Far Infrared (THz) spectrum



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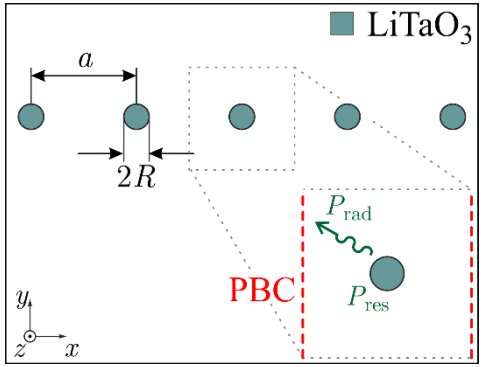
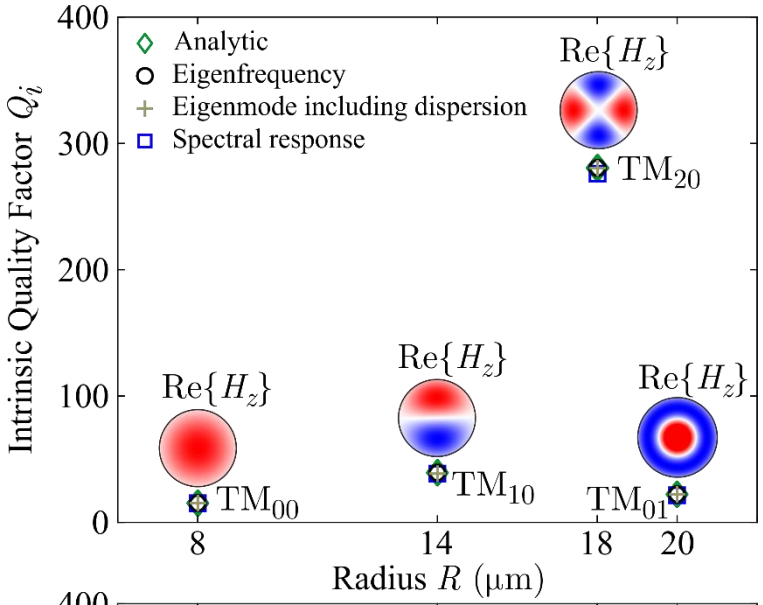
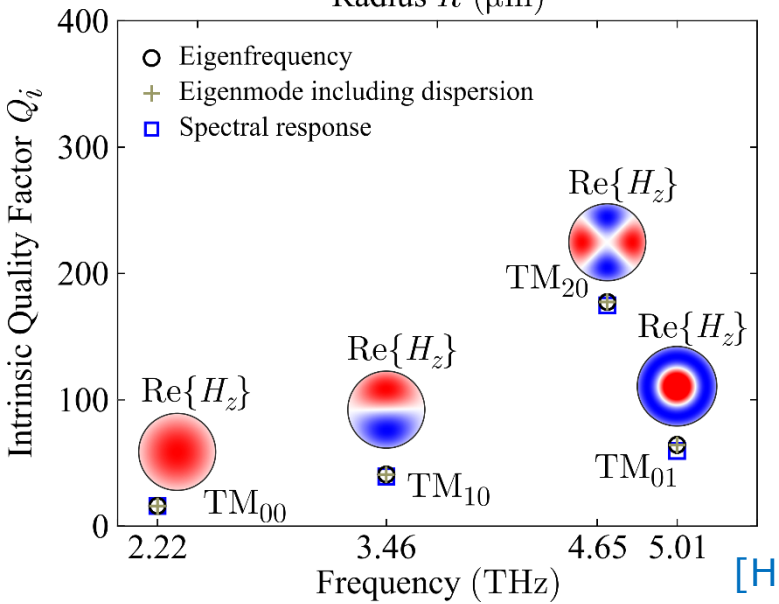
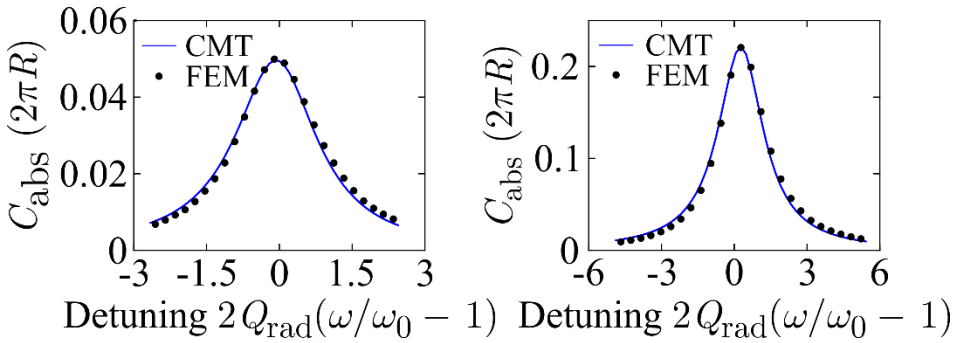


[Tsilipakos, *Adv. Opt. Mater. 6*, 1800633, 2018]







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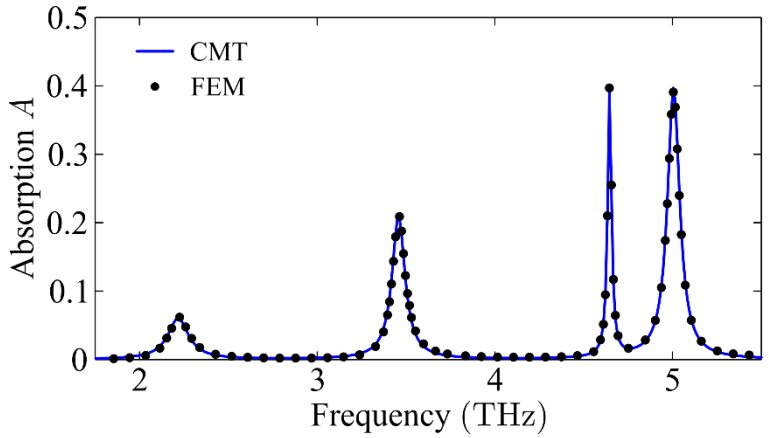
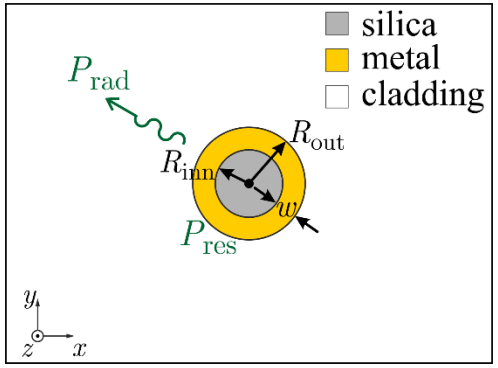


$$A = \sum_k \frac{2(1/Q_{res,k})(1/Q_{rad,k})}{4(\omega_k/\omega_0-1)^2+(1/Q_{res,k}+1/Q_{rad,k})^2}$$

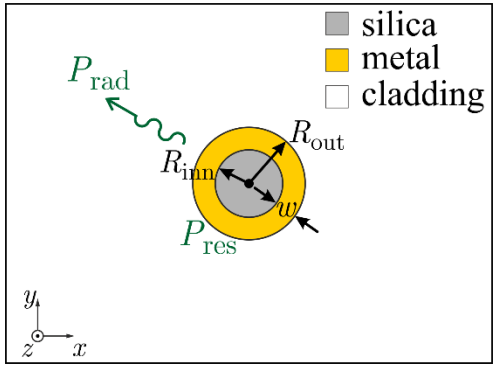
[H. A. Haus, *Waves and Fields in Optoelectronics*]

Plasmonic core-shell @ Visible spectrum



- Strong dispersion
- Radiation <u>and</u> ohmic loss

Plasmonic core-shell @ Visible spectrum



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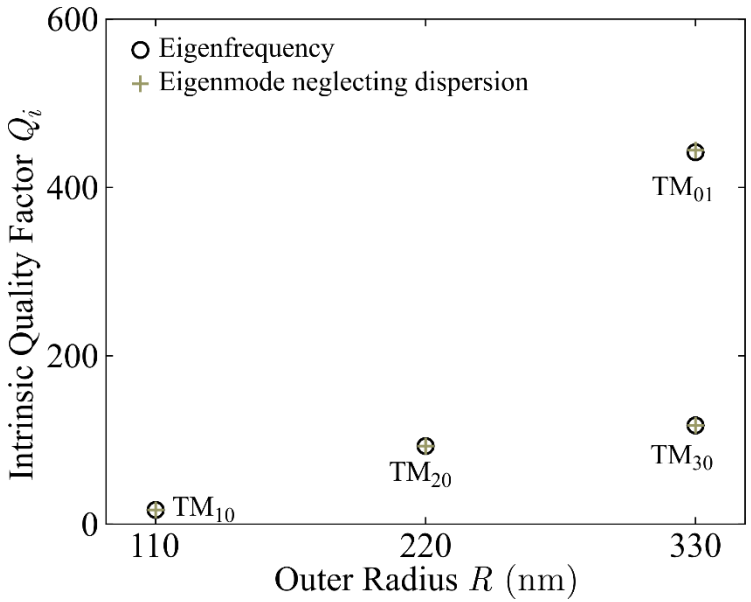

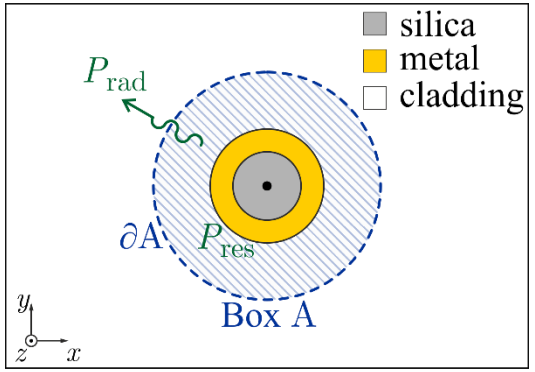

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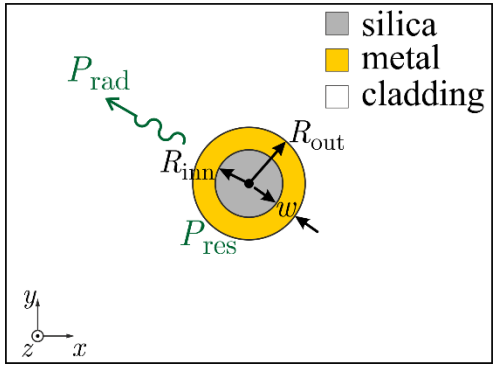

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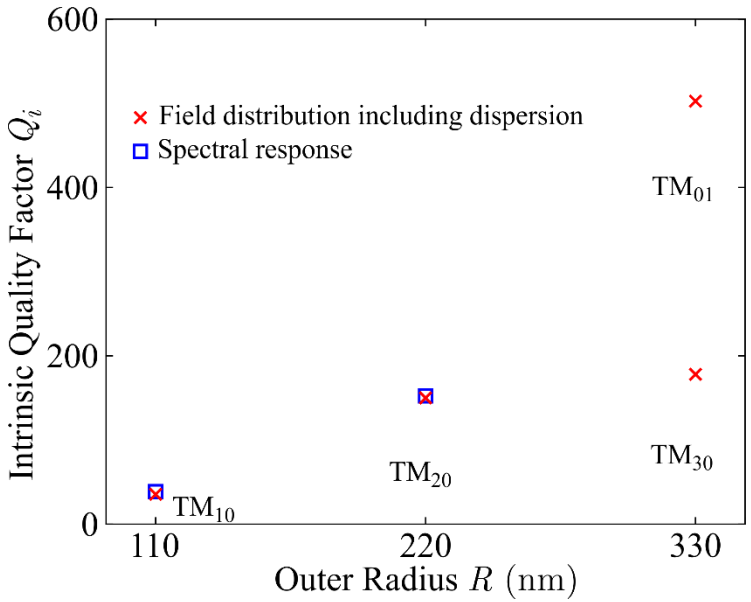

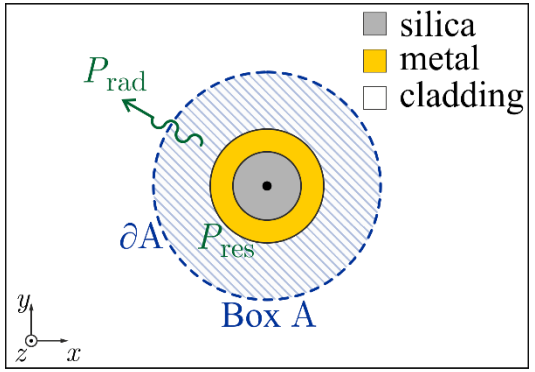

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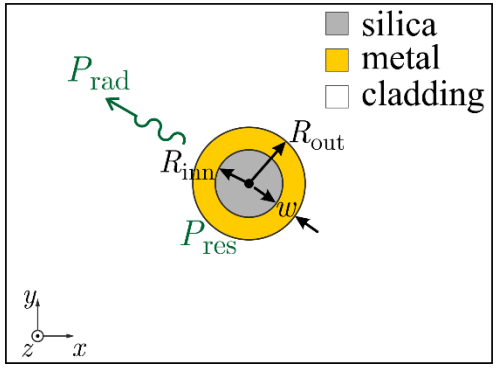

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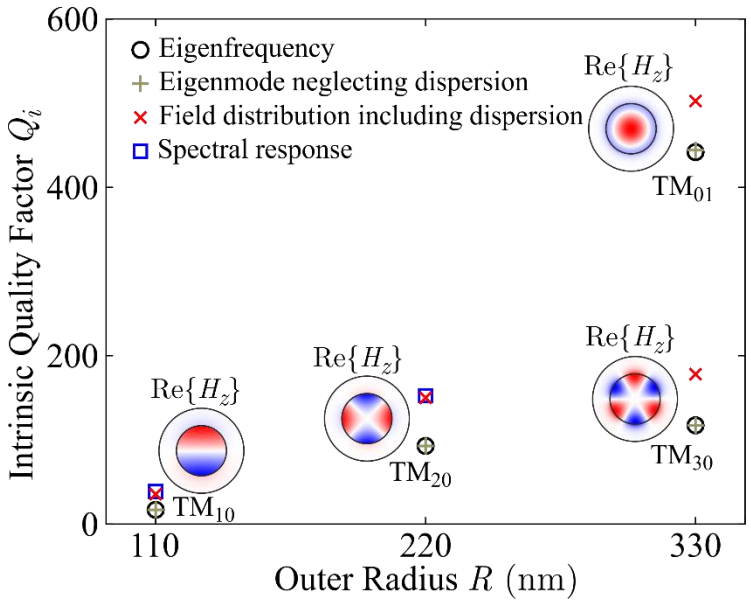
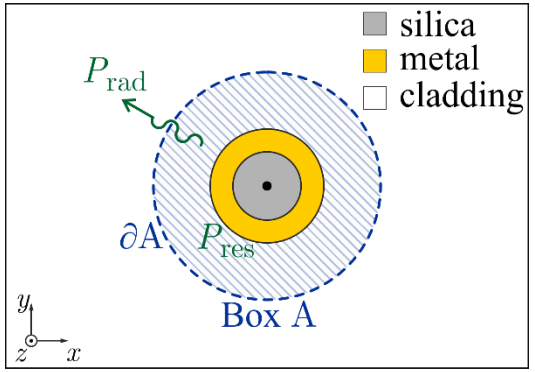
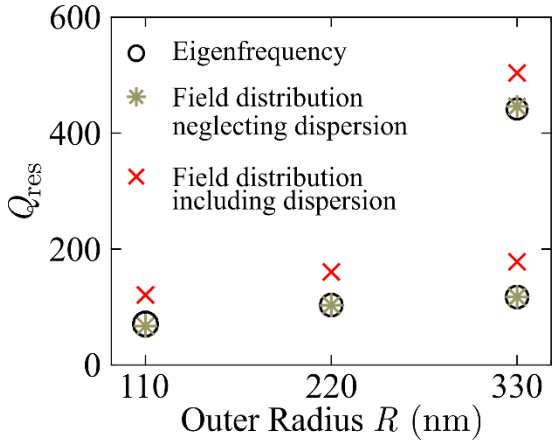
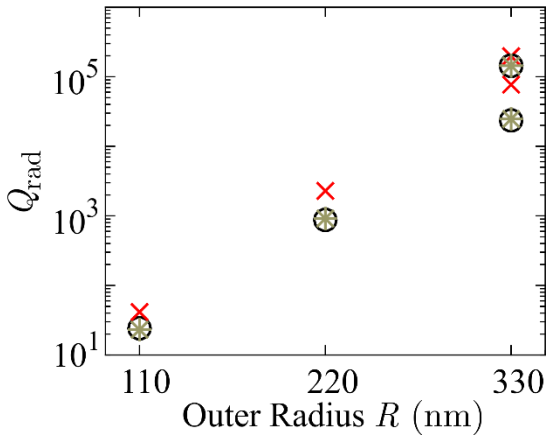
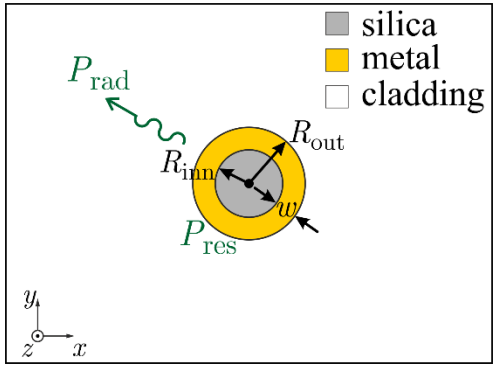




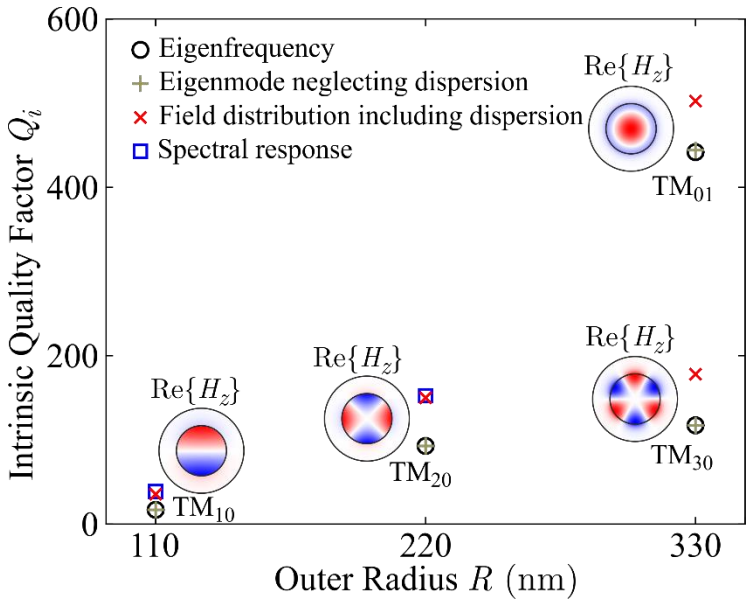


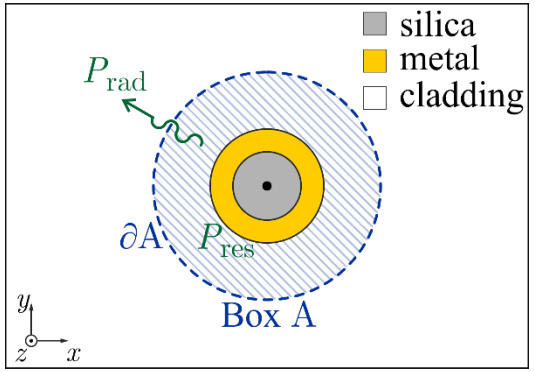


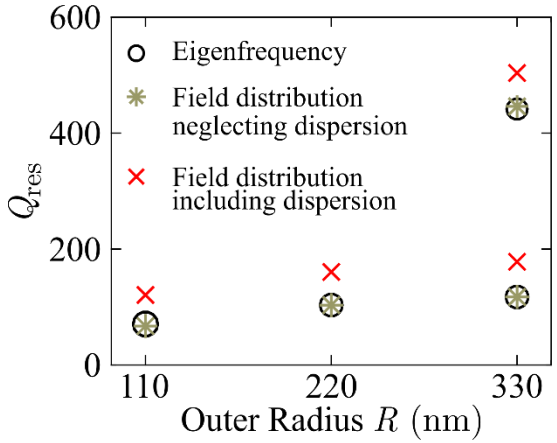
Plasmonic core-shell @ Visible spectrum

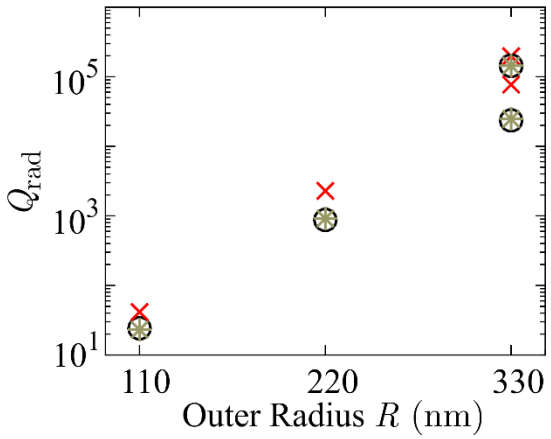


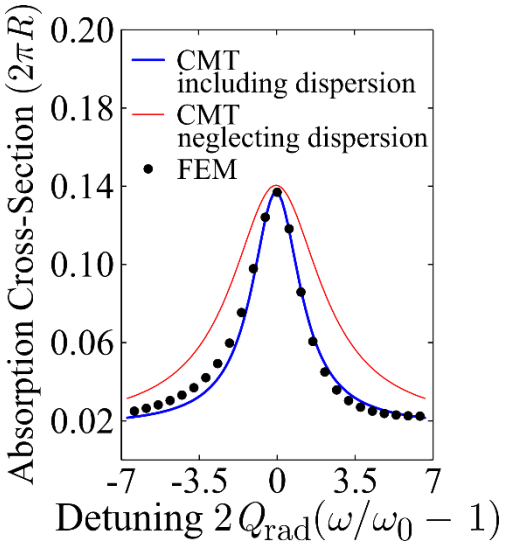
- Strong dispersion
- Radiation and ohmic loss











Conclusion

Summary

- Presentation of the *Q*-factor calculation in contemporary photonic resonant structures
- Evaluation in four judiciously chosen structures
- In depth examination of their applicability range with commercially available software

Results

- Strong influence of dispersion on the correct calculation of *Q*
- Light leakage also hardens the calculation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On the calculation of the quality factor in contemporary photonic resonant structures

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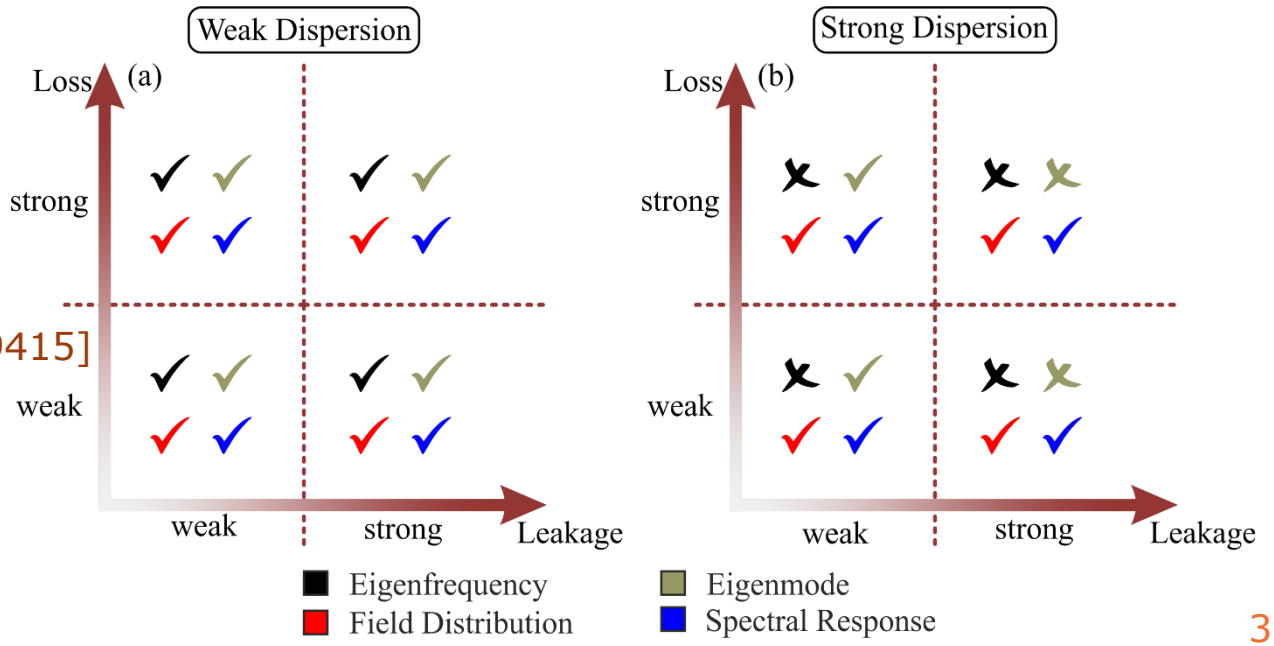
[arXiv:1902.09415]



Thank you!

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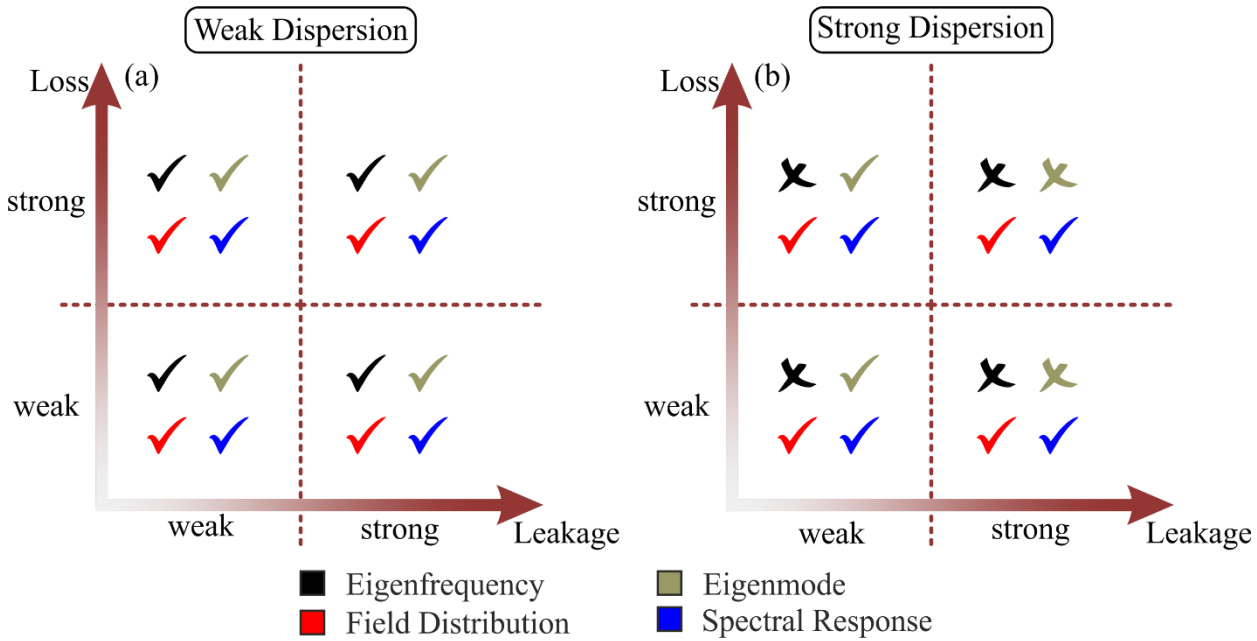

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