Single- and Multi-Channel Nonlinear Effects in Graphene-Enhanced Resonators

T. Christopoulos, O. Tsilipakos, G. Sinatkas, and E. E. Kriezis
Motivation & Main Objectives

Motivation

- Exploit graphene’s nonlinear properties in practical nanophotonic resonators.
- Extend perturbation theory and coupled mode theory (CMT) frameworks for 2-D materials (graphene, transition metal dichalcogenides, black phosphorus).
- Understand the interplay of different nonlinear effects using an accurate and efficient simulation framework.

Main Objectives

- Develop a CMT framework for single- (Optical Bistability) and multi-channel nonlinear effects (Four-Wave Mixing, Saturable Absorption) for graphene-enhanced resonators in NIR and THz.
- Validate the CMT framework against full-wave simulations.
- Allow for the systematic inclusion of various linear and nonlinear effects, including carrier effects in Si.
- Demonstrate that nonlinear optical effects are exploitable at practical power levels by proper engineering.

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Presentation Overview / At a glance

Optical properties of graphene: quick review
- Linear surface conductivity
- Nonlinear surface conductivity tensor

Optical Bistability
- Bistability with TW resonators in the NIR
- Bistability with TW resonators in the FIR

Degenerate Four Wave Mixing
- FWM with SW resonators in the FIR

Saturable Absorption
- Intensity-dependent conductivity model
- SA-induced switching/routing in nanophotonic disk resonators

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Maxwell’s curl equations, including both linear and nonlinear contributions (frequency-domain). 2D materials are introduced as a surface current density:

\[
\nabla \times \mathbf{E} = -j \omega \mu_0 \mathbf{H} \\
\nabla \times \mathbf{H} = +j \omega \varepsilon_0 \mathbf{E} + j \omega \mathbf{P}_{\text{lin}} + \mathbf{J}_{\text{lin}} \delta_s (r) + j \omega \mathbf{P}_{\text{NL}} + \mathbf{J}_{\text{NL}} \delta_s (r)
\]

perturbation terms

Surface current density on graphene (time-domain):

\[
\mathbf{J}_s = \sigma_{s}^{(1)} \mathbf{E} + \sigma_{s}^{(2)} \mathbf{E} \mathbf{E} + \mathbf{E} \mathbf{E} \mathbf{E} + ...
\]

important

Overall current density

\[
\mathbf{J} = \mathbf{J}_s \delta_s (r) = (\mathbf{J}_{\text{lin}} + \mathbf{J}_{\text{NL}}) \delta_s (r)
\]

Graphene sheet

Surface conductivity tensor, rank \((n+1)\), in \([\text{S/(m/V)}^{n-1}]\)

\[
\sigma_{s}^{(n)}
\]

\[
\sigma_{s,xx} = \sigma_{s,zz} \equiv \sigma_1, \quad \sigma_{s,xz} \approx 0
\]
Optical properties of graphene / Linear surface conductivity in the NIR

\[ J_s = J_{s,\text{intra}} + J_{s,\text{inter}} = (\sigma_{\text{intra}} + \sigma_{\text{inter}})E\parallel \]

- **Symmetric** change around Dirac point \((\mu_c = 0)\)
- **Step behavior** for \(\text{Re}\{\sigma}\) at \(E_f = \pm 0.4 \text{ eV}\) for \(\lambda = 1.55 \mu\text{m}\)

\[
\sigma_{\text{intra}}(\mu_c) = \frac{-je^2k_BT}{\pi\hbar^2(\omega - j/\tau_1)} \left[ \frac{\mu_c}{k_BT} + 2\ln(e^{-\mu_c/k_BT} + 1) \right]
\]
\[
\sigma_{\text{inter}}(\mu_c) = \frac{-je^2}{4\pi\hbar} \ln \left[ \frac{2|\mu_c| - \hbar(\omega - j/\tau_2)}{2|\mu_c| + \hbar(\omega - j/\tau_2)} \right]
\]

Hanson, *J. Appl. Phys.* 103, 064302, 2008

\[
\lambda = 1.55 \mu\text{m}
\]

Low loss for Kerr
Optical properties of graphene / Linear surface conductivity in the FIR

- Only intraband transitions allowed: Drude-like response
- Low losses (small $\text{Re}\{\sigma\}$) and plasmonic response (high $\text{Im}\{\sigma\}$)
- Strong dispersion

\[
\sigma_1 = \sigma_{\text{intra}}
\]

\[
\sigma_{\text{intra}}(\mu_c) = \frac{-je^2k_BT}{\pi\hbar^2(\omega - j/\tau_1)} \left[ \frac{\mu_c}{k_BT} + 2\ln(e^{-\mu_c/k_BT} + 1) \right]
\]

Falkovsky, Phys. Usp. 51, 887, 2008
Third-order nonlinear term, which accounts for the Kerr effect, TPA, THG and FWM:

\[ J_{NL,j}(\omega_0) = J_{NL} \cdot \mathbf{j} = \frac{3}{4} \sum_{klm} \sigma^{(3)}_{s,jklm}(\omega_0; -\omega_0, \omega_0) E_k E_l^* E_m \]

\( J_{NL,j}(\omega_0) \) is the j-th Cartesian component of the nonlinear surface current - refers to monochromatic or quasi-monochromatic waves (SVE)

Surface conductivity tensor, 4th-rank (81 elements), units [S/(m/V)^2]

<table>
<thead>
<tr>
<th>Source</th>
<th>Non-zero</th>
<th>Independent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hexagonal 2D crystal</td>
<td>8+6</td>
<td>3+3</td>
</tr>
<tr>
<td>Gorbach et al., OL, 2013</td>
<td>8+6</td>
<td>1+1</td>
</tr>
<tr>
<td>Cheng et al., New J. Phys, 2014</td>
<td>8</td>
<td>2</td>
</tr>
<tr>
<td>Simplest model</td>
<td>8</td>
<td>1</td>
</tr>
</tbody>
</table>

Simplest form (8 non-zero, 1 independent), i.e. 2D-equivalent of an isotropic bulk (3D) medium:

\[ \sigma^{(3)}_{s,jklm} = \frac{1}{3} \left( \delta_{jk} \delta_{lm} + \delta_{jm} \delta_{kl} + \delta_{jl} \delta_{mk} \right) \]

For example, a graphene sheet normal to the z-axis

\[ \sigma_{s,xxx}^{(3)} = \sigma_{s,yyy}^{(3)} = \sigma_{s,mmm}^{(3)} = \frac{\sigma_{s,mmn}^{(3)}}{3} \]

\( \{m, n\} = \{x, y\} \)
Optical properties of graphene / Nonlinear surface conductivity tensor

\[
\sigma^{(3)}_{s, jklm} = \sigma_3 \frac{1}{3} (\delta_{jk} \delta_{lm} + \delta_{jm} \delta_{kl} + \delta_{jl} \delta_{mk})
\]

Nonlinear surface current solely depends on the tangential E-field

\[
J_{NL} = \frac{1}{4} \sigma_3 \left[ 2(E_{0,\|} \cdot E^*_{0,\|})E_{0,\|} + (E_{0,\|} \cdot E_{0,\|})E^*_{0,\|} \right]
\]

- Graphene behaves as a self-defocusing material in the NIR.
- High nonlinearity but the overlap with the optical mode may be weak.
- Graphene behaves as self-focusing material in the FIR.

<table>
<thead>
<tr>
<th>NIR (1550 nm)</th>
<th>Equivalent nonlinear index (n_2) [m²/W]</th>
<th>Surface conductivity (\sigma_3) [S/(m/V)²]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Demeteriou, <em>Opt. Express</em> 24, 13033, 2016</td>
<td>(-0.8 \times 10^{-13})</td>
<td>(-j1.3 \times 10^{-21})</td>
</tr>
<tr>
<td>Dremetsika, <em>Opt. Lett.</em> 41, 3281, 2016</td>
<td>(-1.1 \times 10^{-13})</td>
<td>(-j1.7 \times 10^{-21})</td>
</tr>
<tr>
<td>Vermeulen, <em>Phys. Rev. Applied</em> 6, 044006, 2016</td>
<td>(-1.0 \times 10^{-13})</td>
<td>(-j1.6 \times 10^{-21})</td>
</tr>
<tr>
<td>Alexander, <em>ACS Photonics</em> 5, 4944, 2018</td>
<td>-</td>
<td>(up to) (-j2.0 \times 10^{-19})</td>
</tr>
<tr>
<td>Alexander, <em>ACS Photonics</em> 4, 3039, 2017</td>
<td>-</td>
<td>(up to) (-j4.3 \times 10^{-19})</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>FIR (10 THz)</th>
<th>Equivalent nonlinear index (n_2) [m²/W]</th>
<th>Surface conductivity (\sigma_3) [S/(m/V)²]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mikhailov, <em>J. Phys.-Condens. Matter</em> 20, 384204, 2008</td>
<td>(+2.4 \times 10^{-13})</td>
<td>(+j4.7 \times 10^{-19})</td>
</tr>
</tbody>
</table>

Nonlinear Kerr coefficient \(n_2\) [m²/W]
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Hysteresis Loop (Bistability): develops in systems exhibiting **nonlinearity** and **feedback**

- **Travelling-Wave resonator** → provides the feedback
- **Intensity buildup** inside the resonator → Manifestation of nonlinearities

\[ \sigma^{(3)} \rightarrow \Delta \sigma \propto |E|^2 \rightarrow \Delta \omega_{\text{res}} \]

**Perturbation Theory**

- \( \Delta \omega < 0 \)
- Proportional to energy \( W \)
- Estimated from linear calculations
Optical Bistability / Basic concepts

Nonlinearity + optical feedback → Hysteresis loop (bistability)

Prerequisite:
\[ \omega_{op} < \omega_{res} \text{ if } \Delta \omega_{res} < 0 \]

Mathematical Framework

- Perturbation Theory + Temporal CMT
- Linear Full-Wave Simulations (3D-VFEM)


Increase in transmission

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**Nonlinear frequency shift** with both polarization & current density nonlinearities

\[
\frac{\Delta \omega}{\omega_0} = - \int_V \mathbf{P}_{NL} \cdot \mathbf{E}_0^* dV - j \frac{1}{\omega_0} \int_V \mathbf{J}_{NL} \cdot \mathbf{E}_0^* dV
\]

\[
\int_V \frac{d}{d\omega} \left( \omega \varepsilon_0 \varepsilon_r + \sigma_{1m}^{(1)} \right) \mathbf{E}_0 \cdot \mathbf{E}_0^* dV + \mu_0 \int_V \mathbf{H}_0 \cdot \mathbf{H}_0^* dV
\]

Bulk nonlinear media

Includes the 2D sheet current

Dispersive electric energy & complex conductivity contribution

**Complex nonlinear frequency shift**, accounting both for the Kerr effect and TPA

\[
\Delta \omega = \left[ -\gamma_{b \text{ Kerr}} + j \gamma_{b \text{ TPA}} \right] \mathbf{W}_{\text{res}} = \left[ -\left( \gamma_{b \text{ Kerr}} + \gamma_{s \text{ Kerr}} \right) + j \left( \gamma_{b \text{ TPA}} + \gamma_{s \text{ TPA}} \right) \right] \mathbf{W}_{\text{res}}
\]

\[
\gamma_{b \text{ Kerr}} = 4 \left( \frac{\omega_0}{c_0} \right)^3 \omega_0 c_0 \kappa_{b \text{ Kerr}} n_2^{\text{max}}
\]

Bulk nonlinear feedback parameter

\[
\kappa_{b \text{ Kerr}} = \left( \frac{c_0}{\omega_0} \right)^3 \int_V \frac{d}{d\omega} \left[ \varepsilon_r |\mathbf{E}_0|^2 dV + \frac{\sigma_{1m}^{(1)}}{\varepsilon_0} |\mathbf{H}_0|^2 dV + \frac{1}{\varepsilon_0} \int_S \frac{d}{d\omega} \left[ \varepsilon_r |\mathbf{E}_0|^2 dS + \frac{\sigma_{1m}^{(1)}}{\varepsilon_0} |\mathbf{H}_0|^2 dS \right] \right] n_2^{\text{max}}
\]

\[
\gamma_{s \text{ Kerr}} = \left( \frac{\omega_0}{c_0} \right)^4 \kappa_{s \text{ Kerr}} \sigma_{3,1m}^{\text{max}}
\]

Surface nonlinear feedback parameter

\[
\kappa_{s \text{ Kerr}} = \left( \frac{c_0}{\omega_0} \right)^4 \int_S \sigma_{3,1m} \left( |\mathbf{E}_0|^4 + |\mathbf{E}_0||\mathbf{E}_0|^2 + |\mathbf{E}_0||\mathbf{H}_0|^2 dS \right)
\]

\[
\int_V \frac{d}{d\omega} \left[ \varepsilon_r |\mathbf{E}_0|^2 dV + \frac{\sigma_{1m}^{(1)}}{\varepsilon_0} |\mathbf{H}_0|^2 dV + \frac{1}{\varepsilon_0} \int_S \frac{d}{d\omega} \left| \varepsilon_r |\mathbf{E}_0|^2 dS + \frac{\sigma_{1m}^{(1)}}{\varepsilon_0} |\mathbf{H}_0|^2 dS \right| \right] n_2^{\text{max}}
\]


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Optical Bistability / Coupled Mode Theory (CMT) in time / TW resonators

Steady-state (CW) response

System of polynomial equations

\[
\begin{align*}
    p_{\text{out}} &= [\delta + \text{sgn}(\gamma_{\text{Kerr}})p_i]^2 + (1 - r Q + r_{\text{TPA}}p_i)^2 \\
    p_{\text{in}} &= [\delta + \text{sgn}(\gamma_{\text{Kerr}})p_i]^2 + (1 + r Q + r_{\text{TPA}}p_i)^2 \\
    p_{\text{TPA}} &= r_{\text{TPA}}p_i^2, \quad p_i = p_{\text{in}} - p_{\text{out}} - p_{\text{TPA}}
\end{align*}
\]

Tsilipakos, *J. Lightwave Technol.*, 34, 1333, 2016

\[
\delta = \tau_i (\omega - \omega_0)
\]

Normalized detuning

\[
r Q = Q_i / Q_e
\]

Q-factor ratio

\[
P_0 = \frac{2}{\tau_i^2 |\gamma_{\text{Kerr}}|}
\]

Characteristic power

\[
\tau = 2 Q / \omega_0
\]

\[
\mu = j \sqrt{2 / \tau_e}
\]

\[
|s|^2 \equiv P
\]

\[
|s|^2 = (1 - r Q)(\sqrt{3} + r_{\text{TPA}}) \quad 1 - \sqrt{3} r_{\text{TPA}}
\]

Detuning threshold

\[
\delta_{\text{th}} = \frac{(1 + r Q)(\sqrt{3} + r_{\text{TPA}})}{1 - \sqrt{3} r_{\text{TPA}}}
\]

For appropriate input power & detuning **three real positive solutions → bistability**
Optical Bistability / Graphene Nanoribbon (GNR) TW resonator in FIR frequencies

- GNR ring-resonator side-coupled to a similar bus w/g.
- GNR is formed on a uniform graphene sheet due to a biased uneven Si substrate.
- Optimum mode at 10 THz has $R = 10.1 \mu m$ ($m = 15$) and characteristic power $\sim 76 \mu W$.
- Bistability with **theoretically infinite ER** is possible (at critical coupling) and at $\sim 400 \mu W$ **input power**.
Optical Bistability / Silicon-slot TW resonator in NIR frequencies

- Si-slot w/g ring resonator side-coupled to a similar bus w/g.
- Si-slot w/g overlaid with graphene and TE mode profile.
- Optimum mode has R = 3.25 um and characteristic power 6.1 mW.
- Bistability ignoring free carrier effects (FCEs) in Si. Maximum ER (inf) is for $r_Q = 1.9$ mostly due to TPA in graphene.
Bistability including **free carrier effects in Si** for different carrier lifetimes. For $\tau_c = 50$ ps (through ion implantation or carrier sweeping), $ER = 45$ dB at $P_{in} = 90$ mW.

- **Memory operation**: Super Gaussian set & reset pulses accessing the route $ABA'CA$.

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Degenerate Four Wave Mixing / Basic concepts

**Frequency generation** and **wave mixing** in graphene resonators: multi-channel nonlinear interaction

- Automatic **phase matching**, intensity buildup leading to **high CE**, restrictions in operation frequencies
- **2D materials** $\rightarrow$ infinitesimally thin
- **Dispersion in linear properties** $\rightarrow$ graphene is highly dispersive at the FIR (THz)
- **SPM & XPM** present as well
- **CMT framework** and validation against **full-wave simulations**

Graphene plasmon-polariton standing-wave resonator, directly coupled to two semi-infinite graphene sheets
Nonlinear frequency shift (complex) with both polarization & current density nonlinearities. Allows modeling single-channel effects (SPM, Bistability, Self-Pulsation) and multi-channel (XPM, THG, FWM).

\[
\frac{\Delta \omega}{\omega_0} = -\frac{1}{\omega_0} \int P_{NL} \cdot E_0^* d^d r - i \frac{1}{\omega_0} \int J_{NL} \cdot E_0^* d^{d-1} r
\]

Nonlinear surface current

\[\mathcal{J}_{NL} = \sigma^{(3)} | \mathcal{E} \mathcal{E} \mathcal{E}\]

\[\mathcal{E} = \text{Re}\{E_1 \exp(j \omega_1 t) + E_2 \exp(j \omega_2 t) + E_3 \exp(j \omega_3 t)\}\]

\[J_{NL,\mu}(\omega_k + \omega_\ell + \omega_m) = \frac{1}{4} \sum_{\alpha \beta \gamma} \sigma_{\mu\alpha\beta\gamma}^{(3)} E_k,\alpha E_{\ell,\beta} E_{m,\gamma}\]

\[J_{NL}(\omega_k) = J_{\text{SPM}}(\omega_k) + J_{\text{XPM}}(\omega_k) + J_{\text{DFWM}}(\omega_k)\]

**SPM**
\[
\omega_k = \omega_k - \omega_k + \omega_k
\]

**XPM**
\[
\omega_k = \omega_k - \omega_\ell + \omega_\ell = \omega_k - \omega_m + \omega_m
\]

\[
\omega_1 = -\omega_1 + \omega_2 + \omega_3,
\]

**FWM**
\[
\omega_2 = 2\omega_1 - \omega_3,
\]
\[
\omega_3 = 2\omega_1 - \omega_2
\]

\[J_{\text{DFWM}}(\omega_1) = \frac{1}{4} \sigma_3 \begin{align*}
2(E_{2,||} \cdot E_{3,||})E_{1,||}^* + 2(E_{1,||} \cdot E_{3,||})E_{2,||}^* + 2(E_{1,||}^* \cdot E_{2,||})E_{3,||}^*
\end{align*}\]

\[J_{\text{DFWM}}(\omega_2) = \frac{1}{4} \sigma_3 \begin{align*}
2(E_{1,||} \cdot E_{3,||})E_{1,||}^* + (E_{1,||} \cdot E_{1,||})E_{3,||}^*
\end{align*}\]

\[J_{\text{DFWM}}(\omega_3) = \frac{1}{4} \sigma_3 \begin{align*}
2(E_{1,||} \cdot E_{2,||})E_{1,||}^* + (E_{1,||} \cdot E_{1,||})E_{2,||}^*
\end{align*}\]
Degenerate Four Wave Mixing / Perturbation theory in graphene resonators

**Full nonlinear frequency shift** experienced by the three resonance modes $\omega_1, \omega_2, \omega_3$:

$$\Delta \omega_1 a_1 = -\gamma_{11} |a_1|^2 a_1 - 2 \gamma_{12} |a_2|^2 a_1 - 2 \gamma_{13} |a_3|^2 a_1 - 2 \beta_1 a_1^* a_2 a_3$$
$$\Delta \omega_2 a_2 = -\gamma_{22} |a_2|^2 a_2 - 2 \gamma_{21} |a_1|^2 a_2 - 2 \gamma_{23} |a_3|^2 a_2 - \beta_2 a_1 d_3^*$$
$$\Delta \omega_3 a_3 = -\gamma_{33} |a_3|^2 a_3 - 2 \gamma_{31} |a_1|^2 a_3 - 2 \gamma_{32} |a_2|^2 a_3 - \beta_3 a_1^* a_2^*$$

$$\beta_k = 4 \left( \frac{\omega_k}{c_0} \right)^d \omega_k c_0 \kappa_{k,b}^\text{DFWM} n_2^\text{max} + \left( \frac{\omega_k}{c_0} \right)^{d+1} \kappa_{k,s}^\text{DFWM} \sigma_{3,\text{Im}}^\text{max}$$

$$\kappa_{1,b}^\text{DFWM} = \left( \frac{c_0}{\omega_1} \right)^d \frac{1}{3} \int n_2^2 \left[ 2 (E_1^* \cdot E_3)(E_1^* \cdot E_2) + (E_1^* \cdot E_1^*)(E_2 \cdot E_3) \right] d^dr$$

**Bulk nonlinear feedback parameter**

$$\kappa_{1,s}^\text{DFWM} = \left( \frac{c_0}{\omega_1} \right)^{d+1} \int \sigma_{3,\text{Im}} \left[ 2 (E_{1,||} \cdot E_{3,||})(E_{1,||}^* \cdot E_{2,||}) + (E_{1,||}^* \cdot E_{1,||})(E_{2,||} \cdot E_{3,||}) \right] d^{d-1}r$$

**Surface nonlinear feedback parameter**

Nonlinear coefficients including bulk & surface contributions

$$\gamma_{kl}$$

SPM/XPM

FWM

Christopoulos, Phys. Rev. B, 98, 235421, 2018

**Symmetries in DFWM nonlinear coefficients**

$$\frac{\beta_{1,b}}{\omega_1} = \frac{\beta_{2,b}}{\omega_2} = \frac{\beta_{3,b}}{\omega_3}$$

$$\beta_{1,s} = \beta_{2,s} = \beta_{3,s}$$

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Degenerate Four Wave Mixing / Coupled Mode Theory

\[ \frac{da_1}{dt} = j \left( \omega_1 - \gamma_{11}|a_1|^2 - 2\gamma_{12}|a_2|^2 - 2\gamma_{13}|a_3|^2 \right) a_1 \]

\[ -j2\beta_1 a_1^* a_2 a_3 - \left( \frac{1}{\tau_{i,1}} + \frac{1}{\tau_{e,1}} \right) a_1 + \sqrt{\frac{1}{\tau_{e,1}}} s_{in,1}, \]

\[ \frac{da_2}{dt} = j \left( \omega_2 - 2\gamma_{21}|a_1|^2 - \gamma_{22}|a_2|^2 - 2\gamma_{23}|a_3|^2 \right) a_2 \]

\[ -j\beta_2 a_1^* a_3 - \left( \frac{1}{\tau_{i,2}} + \frac{1}{\tau_{e,2}} \right) a_2 + \sqrt{\frac{1}{\tau_{e,2}}} s_{in,2}, \]

\[ \frac{da_3}{dt} = j \left( \omega_3 - 2\gamma_{31}|a_1|^2 - 2\gamma_{32}|a_3|^2 \right) a_3 \]

\[ -j\beta_3 a_1^* a_2 - \left( \frac{1}{\tau_{i,3}} + \frac{1}{\tau_{e,3}} \right) a_3 + \sqrt{\frac{1}{\tau_{e,3}}} s_{in,3}, \]

Self-phase modulation (SPM)

Cross-phase modulation (XPM)

Degenerate four-wave-mixing (DFWM)

Intrinsic and external cavity loss

Input waves & Output waves

\[ s_{out,k} = \sqrt{\frac{1}{\tau_{e,k}}} a_k \]

- Can be cast in a normalized form introducing a characteristic power level and the frequency detuning with respect to the cavity resonances.
Degenerate Four Wave Mixing / Standing-wave resonator system for FIR frequencies

Graphene plasmon-polariton standing-wave resonator, directly coupled to two semi-infinite graphene sheets

\[ L = 72 \mu m \]

\[ g = 1 \mu m \quad Q_{e,1} = 326, Q_{e,2} = 386, Q_{e,3} = 267 \]

- Calculation of the Q-factors (intrinsic & external) should be done with extra care, see:
  Christopoulos, Opt. Express, 27, 14505, 2019

Due to material (or waveguide) dispersion
\[ \omega_3 \neq 2\omega_1 - \omega_2 \]
Degenerate Four Wave Mixing / Validation of CMT framework against FEM simulations

Full-wave linear simulations (three independent):

Step #1: Linear simulations at $\omega_{1}^{op} = \omega_1, \omega_{2}^{op} = \omega_2$

Step #2: Compute current at $\omega_{3}^{op} = 2\omega_{1}^{op} - \omega_{2}^{op}$, using

$$J(\omega_{3}^{op}) = \frac{1}{4} \sigma_3 \left[ 2(E_{1,\parallel} \cdot E_{2,\parallel}^*)E_{1,\parallel} + (E_{1,\parallel} \cdot E_{1,\parallel}^*)E_{2,\parallel}^* \right]$$

Step #3: Use Step #2 current as source to calculate E-field at $\omega_{3}^{op}$

- Power lost from pump or signal is not accounted and nonlinear interactions between pump-idler and signal-idler are lost.
- This is the undepleted pump case and is accurate for low/medium conversion efficiencies.
Degenerate Four Wave Mixing / Conversion Efficiency (CE)

- **Undepleted pump case:**
  CE monotonically increases (unrealistic)

- **Depleted pump case:**
  \[ \max\{\text{CE}\} = -12.5 \text{ dB} \]
  at \( p_{in,1} = 7.0 \) & \( p_{in,2} = 0.5 \)
  (at the boundary of unstable operation)

- **Full model** (includes SPM/XPM):
  \[ \max\{\text{CE}\} = -31.5 \text{ dB} \]
  at \( p_{in,1} = 1.0 \) & \( p_{in,2} = 0.6 \)

- **Reason for decreased CE:** SPM & XPM shift the cavity resonances away from the operating frequencies, resulting in nonzero detuning and suboptimal light-cavity coupling.

- **Solution:** Pre-shift operating frequencies w.r.t. the unperturbed resonance frequencies to accommodate the red-shift introduced by the Kerr effect (SPM/XPM).

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Degenerate Four Wave Mixing / Conversion Efficiency (CE)

Conversion Efficiency (full-model) at $p_{in,1}=1.0$ & $p_{in,2} = 0.6$

- Pre-shifting greatly improves CE. At $\delta_1=-3.1$ and $\delta_2=-2.35$ CE = -17.6 dB from -31.5 dB. Further fine tuning is possible w.r.t. input power.

- Regions of optical bistability are also predicted, though unfavorable for FWM.

$$\delta_k < -\left(1 + r_{Q,k}\right)^{\sqrt{3}}$$

Christopoulos, Phys. Rev. B, 98, 235421, 2018
Optical properties of graphene: quick review
- Linear surface conductivity
- Nonlinear surface conductivity tensor

Optical Bistability
- Bistability with TW resonators in the NIR
- Bistability with TW resonators in the FIR

Degenerate Four Wave Mixing
- FWM with SW resonators in the FIR

Saturable Absorption
- Intensity-dependent conductivity model
- SA-induced switching/routing in nanophotonic disk resonators
Saturable absorption / Intensity-dependent conductivity model

\[ \sigma_c = \sigma_{\text{intra}} + \sigma_{\text{inter}}(I) \]

\[ \sigma_{\text{inter}} = \sigma_0 \left[ \frac{1}{\sqrt{1 + 3|E_{0,\parallel}|^2/E_{\text{sat}}^2}} + j \frac{1 - \exp\left\{ -\eta_1 \sqrt{|E_{0,\parallel}|^2/E_{\text{sat}}^2} \right\}}{\sqrt{1 + (\eta_1^2|E_{0,\parallel}|^2/E_{\text{sat}}^2)^{0.4}}} \right] \]

- **Intraband conductivity** does not saturate
- **Interband conductivity** saturates and both Re/Im parts are affected
- **Saturation intensity** can be very low \( I_{\text{sat}} = E_{\text{sat}}^2/2\eta_0 \sim 1 \text{ MW/cm}^2 \)

**Coupled Mode Theory** including SA (here excluding SPM/TPA/FCEs):

\[ \frac{da}{dt} = j(\omega_0 + \Delta\omega_{\text{intra}} + \Delta\omega_{\text{inter}})a - \left( \frac{1}{\tau_{\text{SA}}} + \frac{1}{\tau_{\text{intra}}} + \frac{1}{\tau_{\text{rad}}} + \frac{1}{\tau_e} \right) a + j\frac{2}{\tau_e} s_in \]

\[ \Delta\omega_{\text{inter}} = \frac{-1}{4} \int \sigma_0 \frac{1 - \exp\{-\eta_1 \sqrt{|a|^2|E_{\text{ref,||}}|^2/E_{\text{sat}}^2}\}}{\sqrt{1 + (\eta_1^2|a|^2|E_{\text{ref,||}}|^2/E_{\text{sat}}^2)^{0.4}}} |E_{\text{ref,||}}|^2 d^{d-1}r \]

\[ \frac{1}{\tau_{\text{SA}}} = \frac{1}{4} \int \frac{\sigma_0}{\sqrt{1 + 3|a|^2|E_{\text{ref,||}}|^2/E_{\text{sat}}^2}} |E_{\text{ref,||}}|^2 d^{d-1}r \]


http://photonics.ee.auth.gr • Photonics Group, School of ECE, AUTH • Emmanouil Kriezis
Saturable absorption / Ultrafast nanophotonic switching-routing element in the NIR

- Silica-clad silicon disk coupled to Si-wires, with uniform graphene layer on-top. Graphene introduces SA, linear losses and nonlinear frequency shift (Kerr).

- Wavelength #1 is controlled by the high-power wavelength #2. Output can be directed to the “through” or “drop” ports.

- Through Port: Control wavelength provides critical coupling and theoretically allows for infinite ER.

- Drop Port: ER is finite but sufficiently high.

- Control power is ~9 mW and Kerr nonlinearities become important > 10 mW.

Ataloglou, Phys. Rev. A. 97, 063836, 2018
Ataloglou, EOSAM 2018 (Delft)
Saturable absorption / Ultrafast nanophotonic switching-routing element in the NIR

- **IL** of -3 dB, 10 dB **ER** at drop port, 10-50 Gbps rate, some pulse shape **distortion**.
- **Time offset** in control helps early cavity filling and suppresses distortions.

Ataloglou, **EOSAM 2018** (Delft)
Conclusions

- **Single-channel nonlinearities / Bistability:**
  
  Can be exploited both in the photonic (NIR) or plasmonic (FIR) layouts. 
  High Extinction Ratios (ER) and low input power levels are possible. 
  ~400 uW at FIR (10 THz), ~90 mW at NIR (1550 nm).

- **Degenerate Four Wave Mixing:**
  
  High Conversion Efficiencies (CE) at the FIR attributed to the tightly-confined graphene plasmons and the high nonlinear surface conductivity. 
  CE ~ -17 dB, with further prospect for improvement.

- **Saturable Absorption:**
  
  Saturation intensities can be very low and thus SA can become the dominant NL effect. 
  Optically-controlled switching/routing elements are possible in the NIR at ~10 mW control power.

- **Coupled Mode Theory (CMT) framework:**
  
  Highly accurate and flexible – allows systematic incorporation of a broad range of single or multi-channel NL effects. Excellent agreement against FEM simulations for CW conditions.
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Infinite graphene tube (TW resonator) side-coupled to an infinite graphene sheet (w/g): eigenvalue problem (10 THz) and E-field distribution when feeding the bus w/g.

Bistability curves (dispersive & nondispersive case) obtained with CMT and full-wave nonlinear FEM simulations.